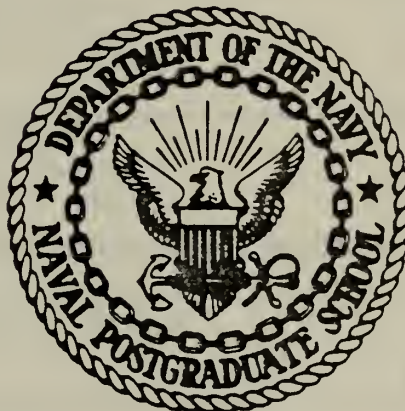


FINITE ELEMENT SOLUTION OF A THREE-
DIMENSIONAL NONLINEAR REACTOR
DYNAMICS PROBLEM WITH FEEDBACK

Eulogio Conception Bermudes

NAVAL POSTGRADUATE SCHOOL

Monterey, California



THESIS

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DYNAMICS PROBLEM WITH FEEDBACK

by

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December 1976

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T177136

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS
BEFORE COMPLETING FORM

1. REPORT NUMBER		2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Finite Element Solution of a Three-Dimensional Nonlinear Reactor Dynamics Problem with Feedback		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis and Mechanical Engineer; Dec 1976	
7. AUTHOR(s) Eulogio Conception Bermudes		6. PERFORMING ORG. REPORT NUMBER	
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		8. CONTRACT OR GRANT NUMBER(s)	
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Naval Postgraduate School Monterey, California 93940		12. REPORT DATE December 1976	
		13. NUMBER OF PAGES 160	
		15. SECURITY CLASS. (of this report) Unclassified	
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.			
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)			
18. SUPPLEMENTARY NOTES			
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)			
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This work examines the three-dimensional dynamic response of a nonlinear fast reactor with temperature-dependent feedback and delayed neutrons when subjected to uniform and local disturbances. The finite element method was employed to reduce the partial differential reactor equation to a system of ordinary differential equations which can be numerically integrated. A program for the numerical solution of large			

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Submitted in partial fulfillment of the
requirements for the degrees of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

and

MECHANICAL ENGINEER

from the

NAVAL POSTGRADUATE SCHOOL
December 1976

Thesis
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ABSTRACT

This work examines the three-dimensional dynamic response of a nonlinear fast reactor with temperature-dependent feedback and delayed neutrons when subjected to uniform and local disturbances. The finite element method was employed to reduce the partial differential reactor equation to a system of ordinary differential equations which can be numerically integrated. A program for the numerical solution of large sparse systems of stiff differential equations developed by Franke and based on Gear's method solved the reduced neutron dynamics equation. Although a study of convergence by refining element mesh sizes was not carried out, the crude finite element mesh utilized yielded the correct trend of neutron dynamic behavior.

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ACKNOWLEDGEMENT

The author wishes to thank Professors Dong H. Nguyen and David Salinas for their moral support and professional guidance throughout this work. Sincere appreciation is also expressed to Professor Dick Franke for his priceless assistance in the implementation of the time integration package and to Professor Giles Cantin for his invaluable counsel on finite elements. Special thanks is given to Ed Donnellan, Kris Butler, and Manus Anderson at the NPS Computer Center for their patience and cooperation in running the long computer programs required by this work. Their willingness to help and their professionalism, I am sure, are recognized and greatly appreciated by NPS students. Finally, I wish to thank my wife, Carmen, who had served outstandingly as mother and father to our children during the course of this study.

I. INTRODUCTION

The nuclear reactor with delayed neutrons and temperature-dependent feedback is a nonlinear system, whose response to uniform and local disturbances differs greatly from that of a linear reactor. The prompt temperature feedback model and one average group of delayed neutrons were incorporated in this work. Practically all neutron dynamics analysis today deals with two-dimensional geometry, implying a symmetry in neutron dynamics behavior during transient [1,2]. This symmetry assumption, however, could be unrealistic in the safety analysis of nuclear reactors. A unique feature of this work is the consideration of the three-dimensional dependence of the neutron flux. No symmetry assumptions were imposed on the problem. This work investigated neutron dynamics under uniform and local disturbances in the core. A uniform initial flux throughout the interior of the reactor was imposed. The finite element method (FEM) was employed to solve this nonlinear initial-boundary value problem. The FEM is quite effective in handling discontinuous forcing functions thereby making it particularly suited for examining the effects of localized perturbations and space-dependent feedbacks.

II. THE NUCLEAR REACTOR WITH TEMPERATURE DEPENDENT FEEDBACK AND DELAYED NEUTRONS

A. THE PHYSICAL SYSTEM

The system under consideration is a fast reactor of cylindrical geometry that is composed of two different regions. The reactor core or region I is cylindrical in shape and is fueled by U-235. Region II or the reflector region completely surrounds the core and is composed of U-238. Both regions were assumed to be homogeneous. Table I lists the physical properties and geometry for each region. A schematic of the fast reactor geometry is shown in figure 1.

Temperature and delayed neutron effects were taken into account. Also, a one-velocity or one-group model was assumed, thereby making the velocity independent of spatial or temporal effects. The delayed neutrons were considered only in region I since region II was assumed to be a non-multiplying medium. The temperature effects were also assumed to be only in the core region.

In general form, the one-velocity neutron diffusion equation is [3]

$$\frac{\partial N(\vec{r}, t)}{\partial t} = vD\nabla^2 N - \Sigma_a vN + S(\vec{r}, t) \quad (1)$$

where $\vec{r} = (x, y, z)$

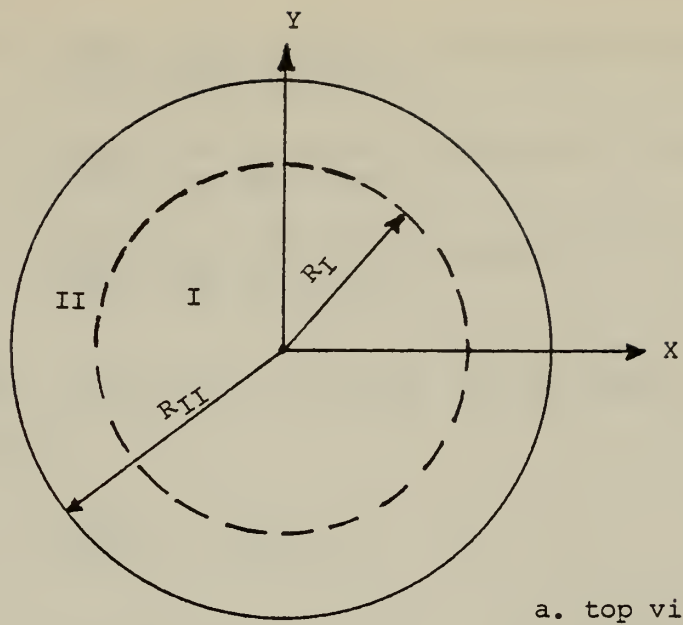
D = neutron diffusion coefficient

Σ_a = macroscopic absorption cross-section

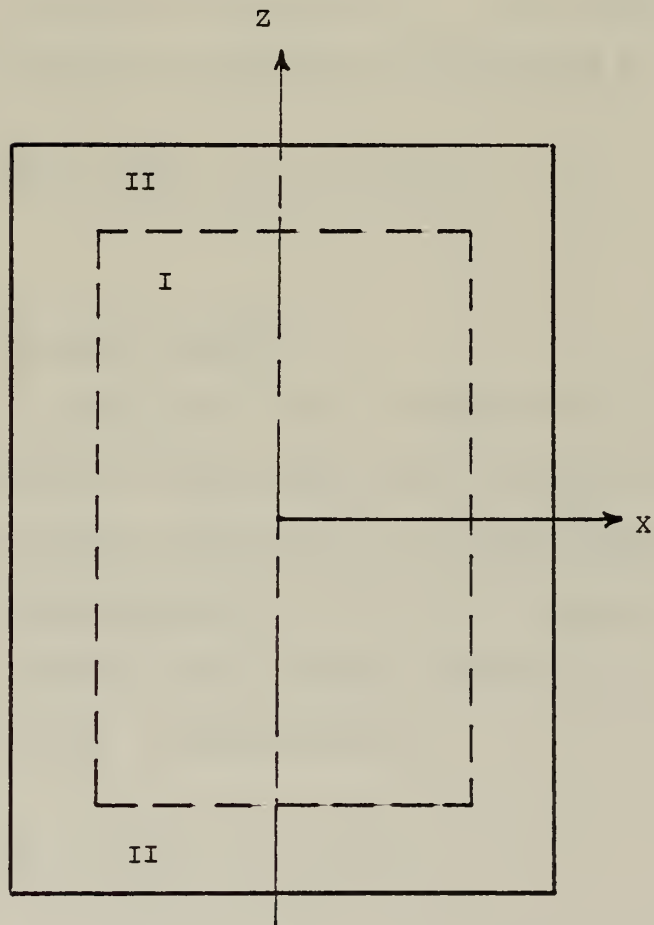
$N(\vec{r}, t)dV$ = number of neutrons in a volume element dV at a point \vec{r} at time t

TABLE I
Physical Constants

<u>SYMBOL</u>	<u>DEFINITION</u>	<u>VALUE</u>
R_I	radius of Region I	60 cm
R_{II}	total reactor radius	90 cm
H_I	height of Region I	160 cm
H_{II}	total reactor height	220 cm
v	neutron speed	4.8×10^7 cm/sec
D_I	core neutron diffusion coefficient	0.913 cm
D_{II}	reflector neutron diffusion coefficient	1.200 cm
Σ_{a_I}	core neutron absorption cross section	0.01401 cm^{-1}
$\Sigma_{a_{II}}$	reflector neutron absorption cross section	0.0040 cm^{-1}
ν	number of neutrons per fission	2.54
Σ_f^*	critical fission cross section	0.005736 cm^{-1}
β	delayed neutron fraction; dollar reactivity	0.00642
ϵ	fission energy	7.652×10^{-12} cal/fission
$\bar{h}(\frac{A}{V})$	modified convection heat transfer coefficient	0.0632 cal/(cm ² sec °C)
α	temperature coefficient	-0.004/°C
$\bar{\lambda}$	abundance-weighted mean decay constant	0.4350 sec^{-1}



a. top view



b. front view

I - core
II - reflector

Figure 1. Schematic of cylindrical reactor.

$vD\nabla^2 N dV$ = number of neutrons diffusing into dV per unit time at time t

$\Sigma_a vN dV$ = number of neutrons absorbed in dV per unit time at time t

$S(\vec{r},t)dV$ = number of neutrons produced in dV per unit time at time t

The neutron number density is related to the neutron flux by the expression [4]

$$\phi(\vec{r},t) = vN(\vec{r},t) \quad (2)$$

where $\phi(\vec{r},t)$ is the flux at time t . The neutron diffusion equation in terms of the flux is depicted by

$$\frac{1}{v} \frac{\partial \phi(\vec{r},t)}{\partial t} = D\nabla^2 \phi - \Sigma_a \phi + S(\vec{r},t) \quad (3)$$

B. PROMPT AND DELAYED NEUTRONS

The source or production term in equation (3) is composed of the contributions of the prompt and delayed neutrons. The majority of the fission neutrons are prompt neutrons that appear almost instantaneously (within 10^{-7} second) on fission. Assuming a fast neutron non-leakage probability of unity, the prompt neutron source is described by

$$S_p(\vec{r},t) = (1-\beta)K_\infty(\vec{r},t) \phi(\vec{r},t) \quad (4)$$

where

$K_\infty(\vec{r},t)$ = infinite medium multiplication factor

β = total fraction of delayed neutrons

The fraction of delayed neutrons is very small (note that $\beta = 0.00642$ from Table I). However, they have a very significant effect on the reactivity because their mean lifetimes are long. Without delayed neutrons, reactor control would not be possible. These delayed neutrons are born in the decay by neutron emission of nuclei produced following the β -decay of certain fission fragments. For example, the β -decay of the fission fragment Br^{87} leads to Kr^{86} plus a neutron. Nuclei such as Br^{87} whose production in fission eventually leads to the emission of a delayed neutron are called delayed neutron precursors [4].

There are six main groups of delayed neutrons. Each group is classified according to its decay constant. The delayed neutron source term is portrayed by

$$S_d(\vec{r}, t) = \sum_{i=1}^6 C_i(\vec{r}, t) \lambda_i \quad (5)$$

where

λ_i = decay constant of the i^{th} group

$C_i(\vec{r}, t)$ = density of the i^{th} precursor

Assuming that the fission fragments do not migrate appreciable distances and assuming a non-circulating fuel reactor [3], the precursor density is delineated by

$$\frac{\partial C_i(\vec{r}, t)}{\partial t} = \beta_i K_{\infty} \Sigma_a \phi - \lambda_i C_i \quad (6)$$

where β_i is the fraction of delayed neutrons of the i^{th}

group. The solution to the precursor equation is in terms of a time integral expressed by

$$C_i(\bar{r}, t) = \beta_i \Sigma_a \int_0^t e^{-\lambda_i(t-t')} K_\infty(\bar{r}, t) \phi(\bar{r}, t) dt' \quad (7)$$

Inserting equation (7) in equation (5) yields the delayed neutron production term as

$$S_d(\bar{r}, t) = \sum_{i=1}^6 \beta_i \lambda_i \Sigma_a \int_0^t e^{-\lambda_i(t-t')} K_\infty(\bar{r}, t) \phi(\bar{r}, t) dt' \quad (8)$$

For convenience the six main groups of delayed neutrons were considered as one group. This was accomplished by using the abundance-weighted mean decay constant defined by

$$\bar{\lambda} = \frac{1}{\beta} \sum_{i=1}^6 \beta_i \lambda_i \quad (9)$$

This is a reasonable approximation since many of the important phenomena of nuclear reactor dynamics can be characterized satisfactorily by combining all the emitters or precursors into one, two, or, at most, three effective groups [3]. Replacing λ_i with the abundance-weighted mean decay constant showed the delayed neutron source term to be

$$S_d(\bar{r}, t) = \beta \bar{\lambda} \Sigma_a \int_0^t e^{-\bar{\lambda}(t-t')} K_\infty(\bar{r}, t) \phi(\bar{r}, t) dt' \quad (10)$$

C. DOPPLER EFFECT AND TEMPERATURE FEEDBACK MODEL

The Doppler effect in fast reactors is due to the temperature broadening of many closely spaced high-energy resonances in both the fission and parasitic-absorption cross-sections.

These resonances basically mean that as the temperature increases, the number of neutrons that are absorbed increases and, similarly, the number of fission events increases.

These nonproductive and productive processes compete in a complicated manner, and the net effect may be either an increase or decrease in reactivity [3]. For most fast reactors, the effect is negative, and it will be so assumed in this work.

The reactivity change with respect to the fuel temperature change is modeled by [2]

$$\frac{dK_{\infty}}{dT} = aT^{-3/2} + bT^{-1} + cT^{(m-1)} \quad (11)$$

where a , b , and c are parameters determined from experimental or neutronic calculations, m is an integer, and T is the current fuel temperature. For most fast reactor cores (ceramic-fueled cores), TdK_{∞}/dT is very close to being constant over a wide range of temperatures. Therefore, it is assumed that a and c are zero and the Doppler coefficient is expressed by

$$b = T \frac{dK_{\infty}}{dT} \quad (12)$$

The following initial conditions were used:

$$K_{\infty}(\bar{r}, 0^+) = K_{\infty}^0 \quad (12a)$$

$$T(\bar{r}, 0^+) = T_0 \quad (12b)$$

The reactivity model is then expressed by

$$K_{\infty}(\bar{r}, t) = K_{\infty}^{\circ} + b \ln \left(\frac{T}{T_0} \right) \quad (13)$$

where

- K_{∞}° = multiplication factor at steady-state
- ν = number of neutrons produced per fission
- T_0 = original fuel temperatures
- b = Doppler constant

The definition of K_{∞}° is

$$K_{\infty}^{\circ} = \frac{\nu \Sigma_f^*}{\Sigma_a} \quad (14)$$

where

- Σ_f^* = critical fission cross-section
- Σ_a^c = macroscopic absorption cross-section of the core

The rise in fuel temperature is depicted by

$$\theta(\bar{r}, t) = T(\bar{r}, t) - T_0(\bar{r}) \quad (15)$$

This is also described by the integral [5]

$$\theta(\bar{r}, t) = \int_0^t f(t-t') \psi(\bar{r}, t) dt' \quad (16)$$

where $f(t-t')$ is the feedback kernel and is dependent upon the type of temperature feedback model used. $\psi(\bar{r}, t)$ is the

rise in neutron flux above the steady state and is expressed by

$$\psi(\vec{r},t) = \phi(\vec{r},t) - \phi_0(r) \quad (17)$$

where $\phi_0(r)$ is the neutron flux at steady state.

There are three types of temperature feedback models that can be considered here. The first is known as "Newton's feedback" which determines the reactor temperature by Newton's law of cooling. The second temperature feedback model is called the adiabatic feedback model. This represents the temperature for the loss of coolant case. The third model is the prompt temperature feedback model in which the fuel temperature follows the behavior of the neutron flux without delay [5,6]. The prompt feedback model was employed in this analysis. The feedback kernel for this temperature feedback model is

$$f(t-t') = \frac{K}{\gamma} \delta(t-t') \quad (18)$$

where K is an energy production operator with units of °C per unit flux and γ , a dimensionless quantity, is related to the mean time for heat transfer to coolant.

Inserting equation (18) in equation (16) and performing the integration produced a rise in temperature of

$$\theta(\vec{r},t) = \frac{K}{\gamma} \psi(\vec{r},t) \quad (19)$$

From equation (19) the temperature of the fuel is

$$T(\bar{r}, t) = \frac{K}{\gamma} \psi(\bar{r}, t) + T_0(\bar{r}) \quad (20)$$

The ratio of the current temperature to the steady-state temperature is therefore

$$\frac{T}{T_0} = \frac{K}{\gamma T_0} \psi + 1 \quad (21)$$

In this work T_0 was considered to be constant throughout the core. Incorporating equation (21) into equation (13) gave the reactivity model of

$$K_\infty(\bar{r}, t) = K_\infty^0 + b \ln \left[\frac{K}{\gamma T_0} \psi(\bar{r}, t) + 1 \right] \quad (22)$$

D. FIELD EQUATIONS

Before establishing the field equations it is desirable to express equation (3) in terms of the rise in flux. Using equation (17) in equation (3) and grouping terms yielded the following diffusion equation:

$$\begin{aligned} \frac{1}{v} \frac{\partial \psi}{\partial t} = & [D \nabla^2 \psi - \Sigma_a \psi + (1-\beta) K_\infty \Sigma_a \psi + \beta \bar{\lambda} \Sigma_a \int_0^t e^{-\bar{\lambda}(t-t')} K_\infty \psi dt'] \\ & + [D \nabla^2 \phi_0 - \Sigma_a \phi_0 + (1-\beta) K_\infty \Sigma_a \phi_0 + \beta \bar{\lambda} \Sigma_a \int_0^t e^{-\bar{\lambda}(t-t')} K_\infty \phi_0 dt'] \end{aligned} \quad (23)$$

The second bracketed term in equation (23) is identically equal to zero since it is the steady state portion of the

diffusion equation. The rise in neutron flux above its steady state value is therefore expressed, for the core, by

$$\begin{aligned} \frac{1}{v} \frac{\partial \psi}{\partial t} = & D \nabla^2 \psi - \Sigma_a \psi + (1-\beta) K_{\infty} \Sigma_a \psi \\ & + \beta \bar{\lambda} \Sigma_a \int_0^t e^{-\bar{\lambda}(t-t')} K_{\infty} \psi dt' \end{aligned} \quad (24)$$

and, for the reflector region, by

$$\frac{1}{v} \frac{\partial \psi}{\partial t} = D \nabla^2 \psi - \Sigma_a \psi \quad (25)$$

Inserting the reactivity model into equation (24) yielded

$$\begin{aligned} \frac{1}{v} \frac{\partial \psi}{\partial t} = & D \nabla^2 \psi - \Sigma_a \psi + (1-\beta) \Sigma_a K_{\infty}^o \psi \\ & + (1-\beta) b \Sigma_a \left\{ \ln \left[\frac{K \psi}{\gamma T_o} + 1 \right] \right\} \psi + \beta \bar{\lambda} K_{\infty}^o \int_0^t e^{-\bar{\lambda}(t-t')} \psi dt' \\ & + \beta \bar{\lambda} \Sigma_a b \left\{ \int_0^t e^{-\bar{\lambda}(t-t')} \left[\ln \left(\frac{K \psi}{\gamma T_o} + 1 \right) \right] \psi dt' \right\} \end{aligned} \quad (26)$$

For the reflector, the last four terms of equation (26) are zero. The effects of the temperature on the delayed neutrons were neglected in this work. The field equations can now be expressed, for the core, as

$$\begin{aligned} \frac{\partial \psi}{\partial t} - v D \nabla^2 \psi + [v \Sigma_a - v(1-\beta) v \Sigma_f] \psi \\ + [-(1-\beta) v \Sigma_a b] \left[\ln \left(\frac{K \psi}{\gamma T_o} + 1 \right) \right] \psi \\ + [-\beta \bar{\lambda} v \Sigma_f v] \left[\int_0^t e^{-\bar{\lambda}(t-t')} \psi dt' \right] = 0 \end{aligned} \quad (27)$$

and, for the reflector, as

$$\frac{\partial \psi}{\partial t} - v D \nabla^2 \psi + v \Sigma_a \psi = 0 \quad (28)$$

The non-linear terms of the core field equation will be linearized accordingly. In more compact form, equations (27) and (28) became

$$\frac{\partial \psi}{\partial t} - c1 \nabla^2 \psi + c2 \psi + c4 \left[\ln \left(\frac{K\psi}{\gamma T_0} + 1 \right) \right] \psi + c5 \left[\int_0^t e^{-\bar{\lambda}(t-t')} \psi dt' \right] = 0 \quad (29)$$

and

$$\frac{\partial \psi}{\partial t} - c1 \nabla^2 \psi + c2 \psi = 0 \quad (30)$$

where the meanings of the coefficients $c1$, $c2$, etc., are obvious for the core and reflector.

Equations (29) and (30) were subjected to the following conditions:

boundary condition:

$$\psi_R(\bar{r}_B, t) = 0 \quad (31)$$

where \bar{r}_B are coordinates of points on the outer surface of the reflector and the subscript R refers to the reflector

continuity of flux:

$$\psi_R(\bar{r}, t) = \psi_c(\bar{r}_I, t) \quad (32)$$

where \bar{r}_I are coordinates of points on the core-reflector interface and the subscript c refers to the core.

III. FINITE ELEMENT

A. INTRODUCTION

The application of the finite elements of nonlinear continua has been mostly in the field of solid mechanics. Prior work [1] using the finite elements of nonlinear continua on a nonlinear reactor dynamics problem has been successful. The FEM in this work was utilized to reduce a nonlinear partial differential equation of the nuclear reactor to a system of nonlinear ordinary differential equations in time. The time integration was accomplished by using a computer program for the numerical solution of stiff differential equations developed by Franke [7]. In order to minimize computer storage requirements, an optimum compacting scheme (OCS), described by Ref. 8, was adopted.

The finite element models of operator equations are generally classified into three categories: a) the variational finite element models such as the Ritz method, b) the weighted residuals method such as the method of Galerkin, and c) the direct finite element models which are not based on functional minimization. From experience in structural mechanics, the most effective method for generating acceptable finite element models of nonlinear equations is the Galerkin method [1]. This work adopted the method of Galerkin in a finite element approximation over the spatial domain of the field equations.

B. THE METHOD OF GALERKIN

The Galerkin method is a special case of the method of weighted residuals. It involves a rational choice of weighting function that is consistent with the type of finite element approximation considered. Indeed, the weighting functions chosen are the basis or shape functions employed in the finite element approximation. There are two favorable characteristics of the Galerkin method which makes it attractive. The first attribute is its amenability to integration by parts. This supplied the freedom of using a lower order finite element than might be otherwise possible. The second favorable characteristic of the Galerkin method is that the symmetric operators in the field equations transform into symmetric matrix operators. Both these attributes are attractive for computational purposes.

Consider the initial-boundary-value problem

$$\frac{\partial \psi}{\partial t} = \mathcal{L}\psi - f(\bar{r}, t) \quad (33)$$

where \mathcal{L} contains the nonlinear operators. According to the spatial finite element discretization, the solution of equation (33) is in the form of an union of an \bar{N} -term approximation given by

$$\psi(\bar{r}, t) \approx \tilde{\psi}(\bar{r}, t) = \bigcup_{j=1}^{\bar{N}} \psi_j(t) G_j(\bar{r}) , \quad j=1, 2, \dots, \bar{N} \quad (34)$$

where \bar{N} is the number of system degrees of freedom (i.e., number of coordinates), and $G_j(\bar{r})$ are the system or global basis functions which span the space of the approximate

solution $\tilde{\psi}(\bar{r}, t)$ [1]. The Einstein summation is used. The global basic functions are "pyramid functions", each of which has a prescribed functional description over a sub-domain of the system and is zero elsewhere [9]. The unknown coordinate functions $\psi_j(t)$ are the time-dependent magnitudes of the approximated flux $\tilde{\psi}(\bar{r}, t)$ and/or its derivatives at discrete nodal points [10].

The residual function, $R(\bar{r}, t)$, is defined such that it is identical to zero when $\tilde{\psi}(\bar{r}, t)$ is equal to the exact solution. The residual function is expressed by

$$R(\bar{r}, t) = \frac{\partial \tilde{\psi}}{\partial t} - \mathcal{L} \tilde{\psi} - f \quad (35)$$

The Galerkin orthogonality condition (using the basis functions as weight functions), when applied to the residual function, requires that

$$\int_{Vol} G_I R(\bar{r}, t) dVol = 0, \quad I=1, 2, \dots, \bar{N} \quad (36)$$

From the field equations, the residual function for the core is

$$\begin{aligned} R(\bar{r}, t) = & \frac{\partial \tilde{\psi}}{\partial t} - c_1 \nabla^2 \tilde{\psi} + c_2 \tilde{\psi} + c_4 \left[\ln \left(\frac{K \tilde{\psi}}{\gamma T_0} + 1 \right) \right] \tilde{\psi} \\ & + c_5 \left[\int_0^t e^{-\bar{\lambda}(t-t')} \tilde{\psi} dt' \right] \end{aligned} \quad (37)$$

and for the reflector

$$R(\bar{r}, t) = \frac{\partial \tilde{\psi}}{\partial t} - c_1 \nabla^2 \tilde{\psi} + c_2 \tilde{\psi} \quad (38)$$

Using equation (34) and applying Galerkin's orthogonality condition produced a core equation of

$$\begin{aligned}
 \int_{Vol} G_I \{ G_J \dot{\psi}_J(t) - \nabla^2 G_J(c1 \cdot \psi)_J + G_J(c2 \cdot \psi)_J \\
 + [\ln(\frac{K}{\gamma T_0} G_K \psi_K + 1)] G_J(c4 \cdot \psi)_J \\
 + [\int_0^t e^{-\bar{\lambda}(t-t')} (c5 \cdot \psi)_J dt'] G_J \} = 0 \quad (39)
 \end{aligned}$$

where $I = 1, 2, \dots, \bar{N}$
 $J = 1, 2, \dots, \bar{N}$
 $K = 1, 2, \dots, \bar{N}$

The reflector has a similar equation without the nonlinearity and is expressed by

$$\int_{Vol} G_I \{ G_J \dot{\psi}_J(t) - \nabla^2 G_J(c1 \cdot \psi)_J + G_J(c2 \cdot \psi)_J \} dVol = 0 \quad (40)$$

C. THE ELEMENT

A three-dimensional quadratic isoparametric element was employed in this work. The parent element is a triangular prism or solid wedge with straight sides. The element shape functions are expressed in terms of area coordinates in the plane of the triangle and by an isoparametric coordinate

along the prism axis. Figure 2 shows the parent element. This element was chosen because of the ease with which it fits the cylindrical structure when it is transformed into a curved element. This type of element has been used before as filler elements [11].

The area coordinates are defined by area ratios. Consider the triangle shown in figure 3. An arbitrary point P within the triangle defines three subareas designated by A_1 , A_2 , and A_3 . The ratio of each of the subareas to the total area is known as an area coordinate. In equation form, the area coordinates are

$$L_1 = A_1/A \quad (41a)$$

$$L_2 = A_2/A \quad (41b)$$

$$L_3 = A_3/A \quad (41c)$$

where A is total area of the triangle. L_1 , L_2 , and L_3 are the natural coordinates for a triangle. The requirement that the sum of the subareas be equal to the total area is obviously satisfied by the identity

$$L_1 + L_2 + L_3 = 1 \quad (42)$$

In the plane of the triangle, only two of the area coordinates are independent.

The isoparametric coordinates are best visualized by considering the rectangular prism shown in figure 4. Isoparametric coordinates are normalized coordinates such that their values on the faces of the rectangle are ± 1 . The $\xi\eta\zeta$ axes are in general not orthogonal. They are orthogonal

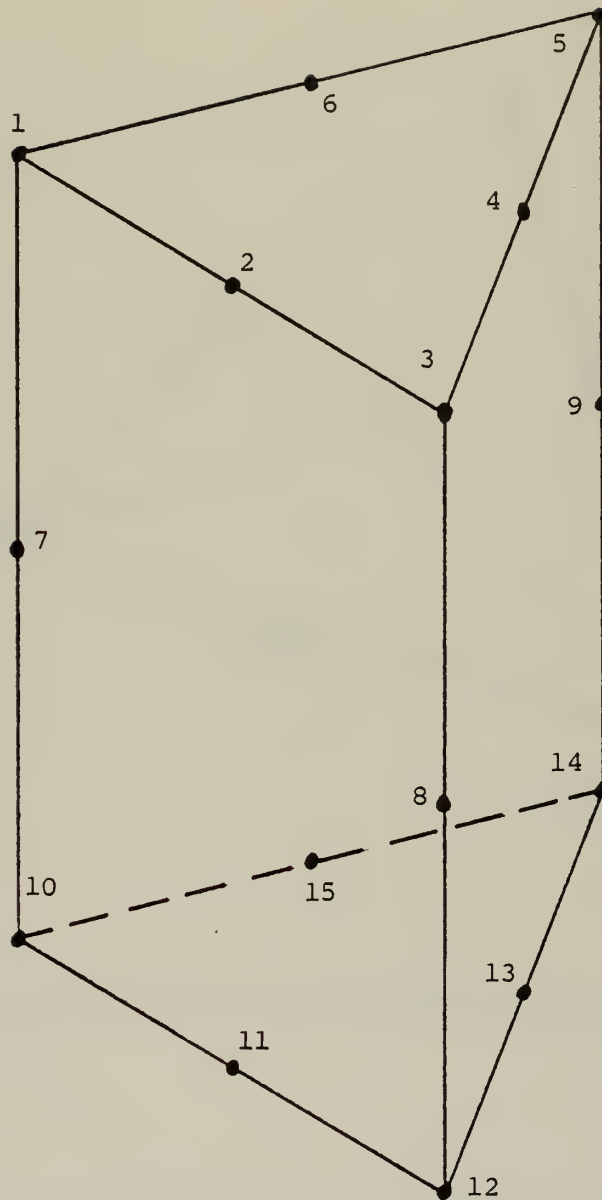


Figure 2. Quadratic triangular prism parent element

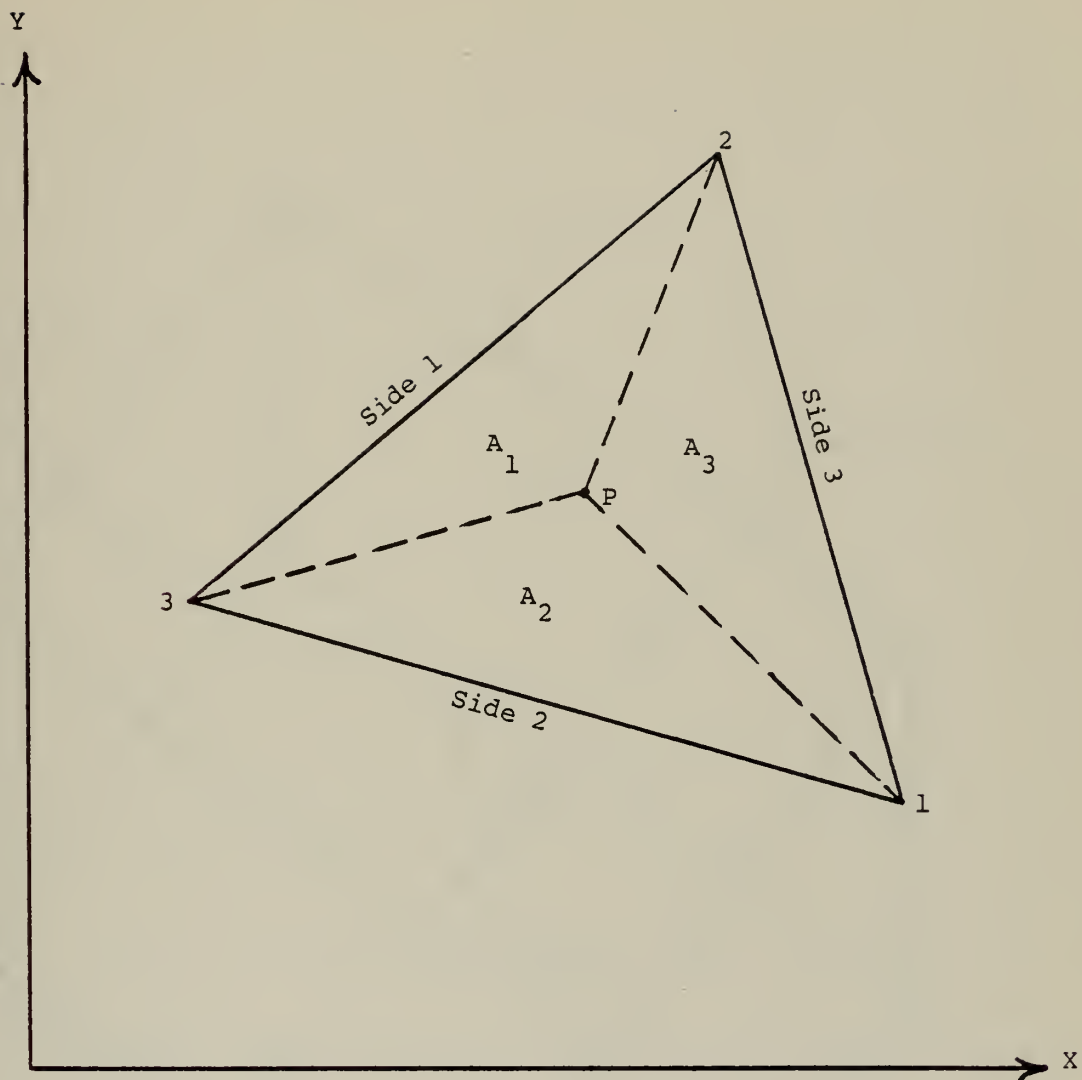


Figure 3. Definition of area coordinates

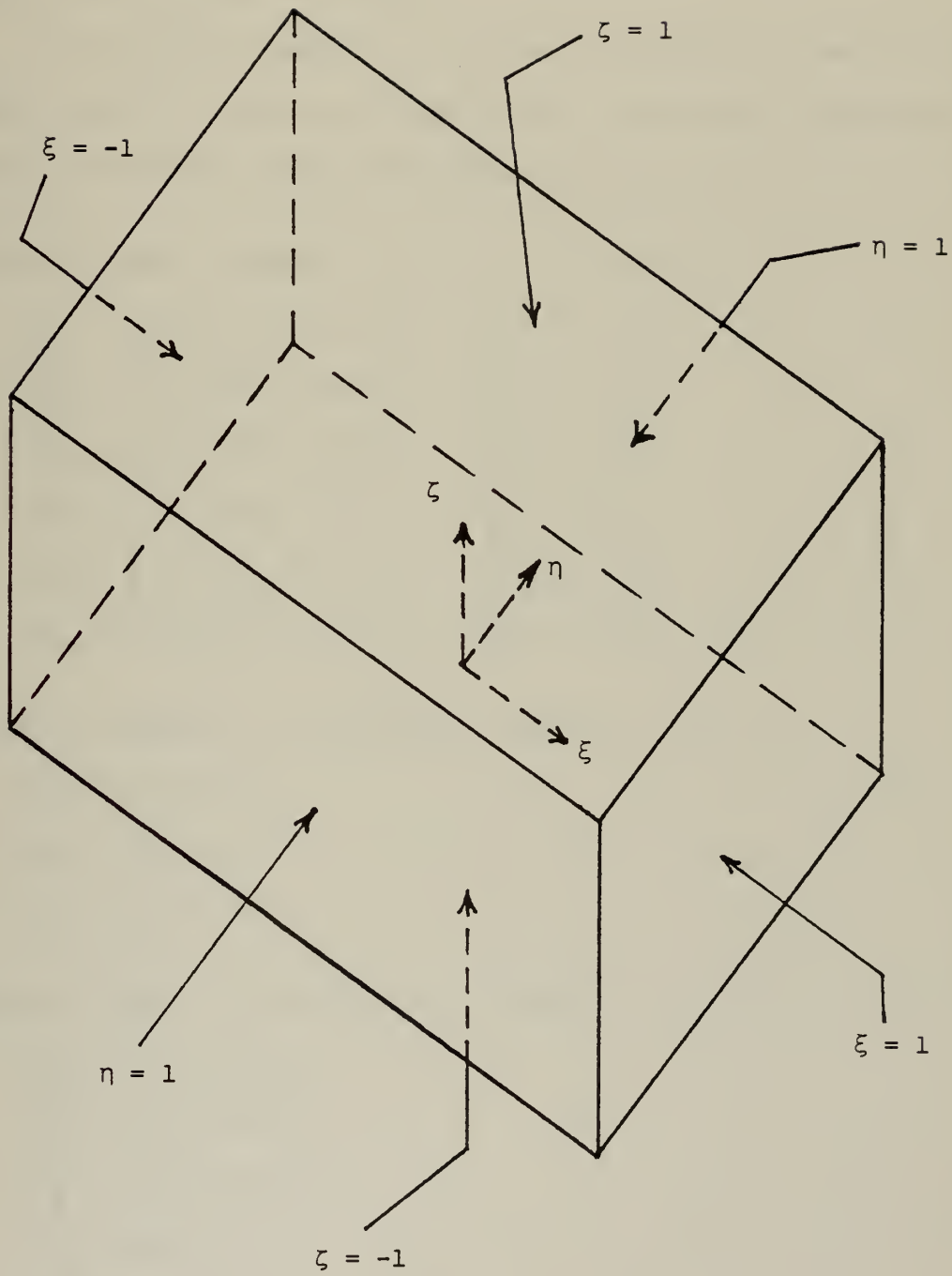


Figure 4. Isoparametric coordinates

only in the special case of a rectangular prism element [12]. The element basis functions use the ζ coordinate in the prism axis and the L_1 , L_2 , and L_3 coordinates in the plane of the triangle.

The parent element, as shown in figure 2, has 15 local nodal points around its periphery. As such, there are 15 basis functions which are given below [11]:

Corner nodes: (nodes 1, 3, 5, 10, 12, 14)

$$N_1 = \frac{1}{2} L_1 (2L_1 - 1)(1 + \zeta) - \frac{1}{2} L_1 (1 - \zeta^2)$$

$$N_3 = \frac{1}{2} L_2 (2L_2 - 1)(1 + \zeta) - \frac{1}{2} L_2 (1 - \zeta^2)$$

$$N_5 = \frac{1}{2} L_3 (2L_3 - 1)(1 + \zeta) - \frac{1}{2} L_3 (1 - \zeta^2)$$

$$N_{10} = \frac{1}{2} L_1 (2L_1 - 1)(1 - \zeta) - \frac{1}{2} L_1 (1 - \zeta^2)$$

$$N_{12} = \frac{1}{2} L_2 (2L_2 - 1)(1 - \zeta) - \frac{1}{2} L_2 (1 - \zeta^2)$$

$$N_{14} = \frac{1}{2} L_3 (2L_3 - 1)(1 - \zeta) - \frac{1}{2} L_3 (1 - \zeta^2)$$

Midside nodes of rectangles: (nodes 7, 8, 9)

$$N_7 = L_1 (1 - \zeta^2)$$

$$N_8 = L_2 (1 - \zeta^2)$$

$$N_9 = L_3 (1 - \zeta^2)$$

Midside nodes of triangles: (nodes 2, 4, 6, 11, 13, 15)

$$N_2 = 2L_1 L_2 (1 + \zeta)$$

$$N_4 = 2L_2 L_3 (1 + \zeta)$$

$$N_6 = 2L_3 L_1 (1 + \zeta)$$

$$N_{11} = 2L_1 L_2 (1 - \zeta)$$

$$N_{13} = 2L_2 L_3 (1 - \zeta)$$

$$N_{15} = 2L_3 L_1 (1 - \zeta)$$

The coordinates of each local node, in terms of L_1 , L_2 , L_3 , and ζ are listed in table II. These element basis functions $\langle N \rangle$ define the geometry of the element. Note that they satisfy the relationship

$$N_i = \begin{cases} 1, & \text{at node } i \\ 0, & \text{at other nodes} \end{cases} \quad (43)$$

On the element level, the variation of the unknown function ψ is approximated by

$$\tilde{\psi}^e = \langle N' \rangle \{ \psi \}^e \quad (44)$$

where $\langle N' \rangle$ = row vector of element shape functions
 $\{ \psi \}^e$ = column vector of time-dependent nodal values of $\tilde{\psi}^e$

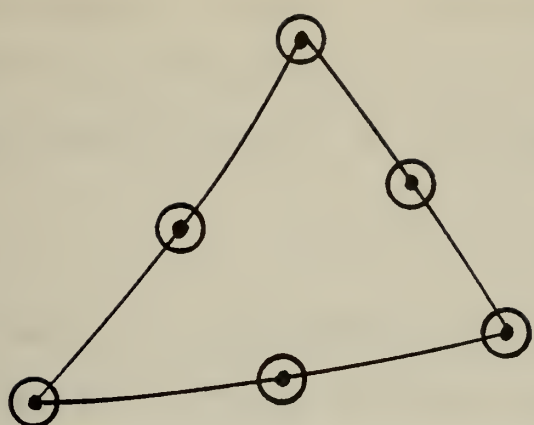
To satisfy continuity requirements, the shape functions $\langle N' \rangle$ have to be such that the continuity of the unknown function ψ is preserved in the parent coordinates [11].

The shape functions $\langle N \rangle$ which characterize the element geometry and the shape functions $\langle N' \rangle$ which describe the unknown function do not necessarily have to be the same. There is no requirement that the nodal values be associated with the same nodes which were used to define the element geometry, though in practice it is often the case. Consider for example, the illustrations in figure 5. If the nodes defining the element geometry and the nodes defining the

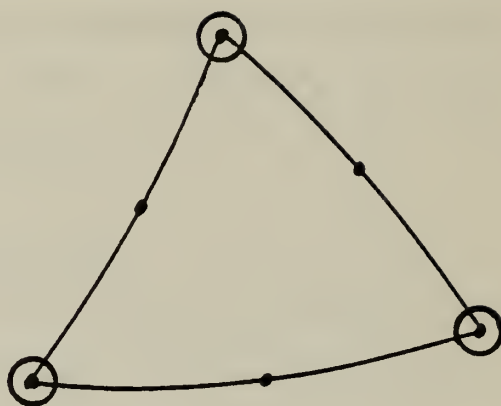
TABLE II

Coordinates of Local Nodal Points

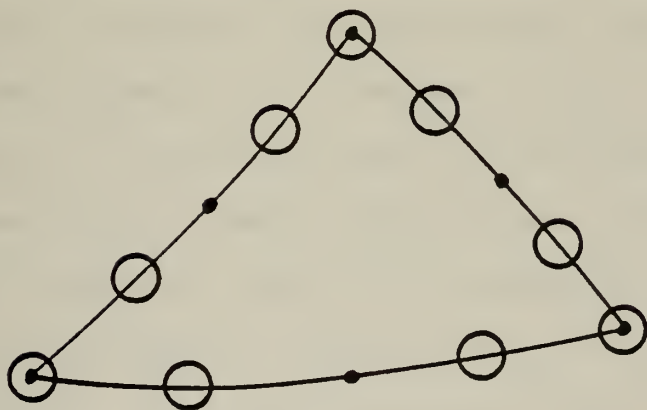
Local Node	L_1	L_2	L_3	5
1	1	0	0	1
2	$\frac{1}{2}$	$\frac{1}{2}$	0	1
3	0	1	0	1
4	0	$\frac{1}{2}$	$\frac{1}{2}$	1
5	0	0	1	1
6	$\frac{1}{2}$	0	$\frac{1}{2}$	1
7	1	0	0	0
8	0	1	0	0
9	0	0	1	0
10	1	0	0	-1
11	$\frac{1}{2}$	$\frac{1}{2}$	0	-1
12	0	1	0	-1
13	0	$\frac{1}{2}$	$\frac{1}{2}$	-1
14	0	0	1	-1
15	$\frac{1}{2}$	0	$\frac{1}{2}$	-1



(a) Isoparametric



(b) Super-parametric



(c) Sub-parametric

- - geometry nodes
- - variable function nodes

Figure 5. Element classification

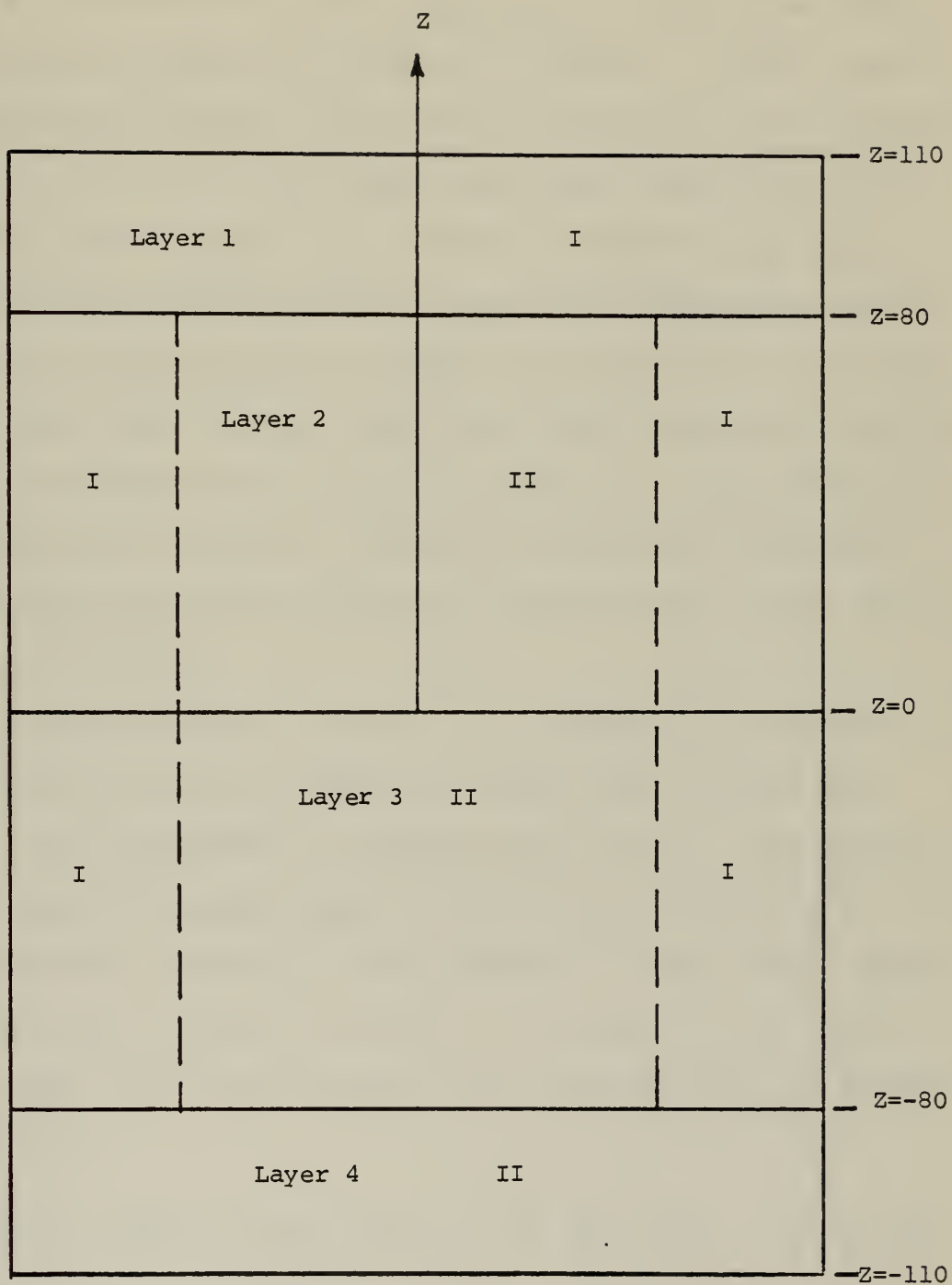
unknown function are identical, the element is known as an isoparametric element. This means that the shape functions describing the geometry and the shape functions describing the unknown function ψ are equal, or

$$\langle N' \rangle = \langle N \rangle \quad (45)$$

If there are more nodes defining the geometry than nodes defining the variable function, the element is called a superparametric element. Using more nodal points to define the unknown function than to describe the geometry of the element results in a subparametric element [11]. This work utilized the isoparametric element classification.

D. DIVISION OF THE SYSTEM INTO ELEMENTS

In three-dimensional space, the division of the system into discrete elements is difficult to visualize. It is virtually impossible to show every nodal point of the system in one schematic. To present a clear view of the discretized domain, a "layer" approach was adopted. The first finite element grid or mesh employed here consisted of 128 elements. Under this grid (mesh I), the reactor was divided into four layers as shown in figure 6. Each layer was composed of 32 elements. The first and fourth layers were each 30 cm in height and each contained entirely reflector elements. The second and third layers were each 80 cm in height and together they encompassed the entire core plus the remaining reflector elements. Each layer, in turn, was partitioned into three horizontal (xy) planes. The top plane included all the global



I - core II - reflector

Figure 6. Layers of mesh I

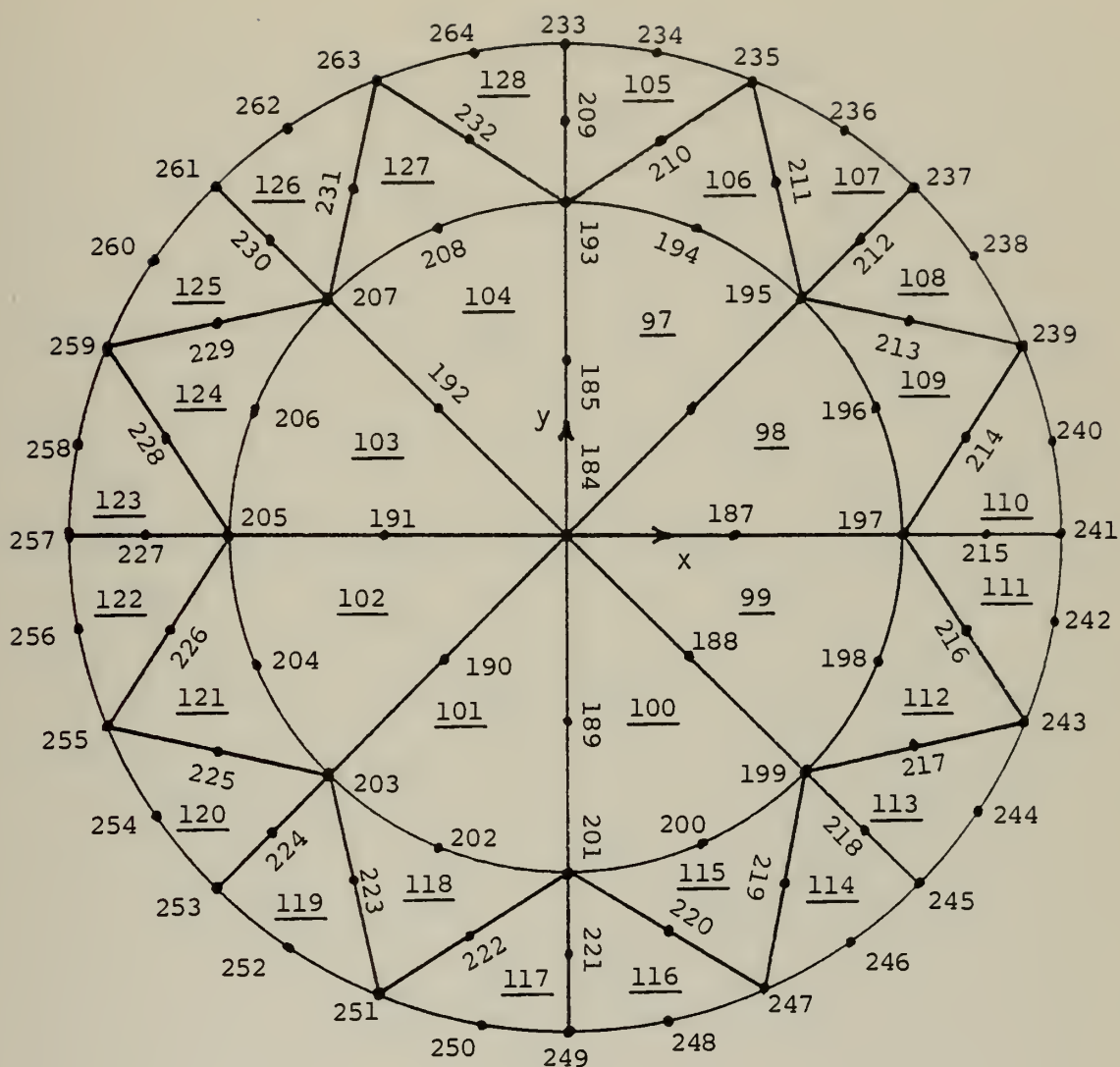
or system nodes corresponding to local or element nodes 1 through 6. The middle plane contained all the system nodes corresponding to the local nodes 7, 8, and 9. System nodes corresponding to element nodal points 10 through 15 comprised the bottom plane. To fix ideas, the three nodal planes of the first layer of mesh I are shown in figures 7, 8, and 9.

In this work there was only one curved side which was in the plane of the triangle, as shown in figure 10. The first, seventh, and tenth element nodes were each arbitrarily assigned to have as an opposite side the curved side of the triangle. The remaining local nodes in each of the respective planes of the element were then numbered consecutively in the counter-clockwise direction.

At this point it is appropriate to define connectivity. The connectivity of an element is a row vector array that relates the local nodes to system nodal points. The connectivity lists the system nodal points that are "connected" to form the element domain in the sequence of local nodal numbering. Thus, the connectivity matrix for mesh I is a matrix of size 128×15 . To illustrate, the connectivity of element number 106 is

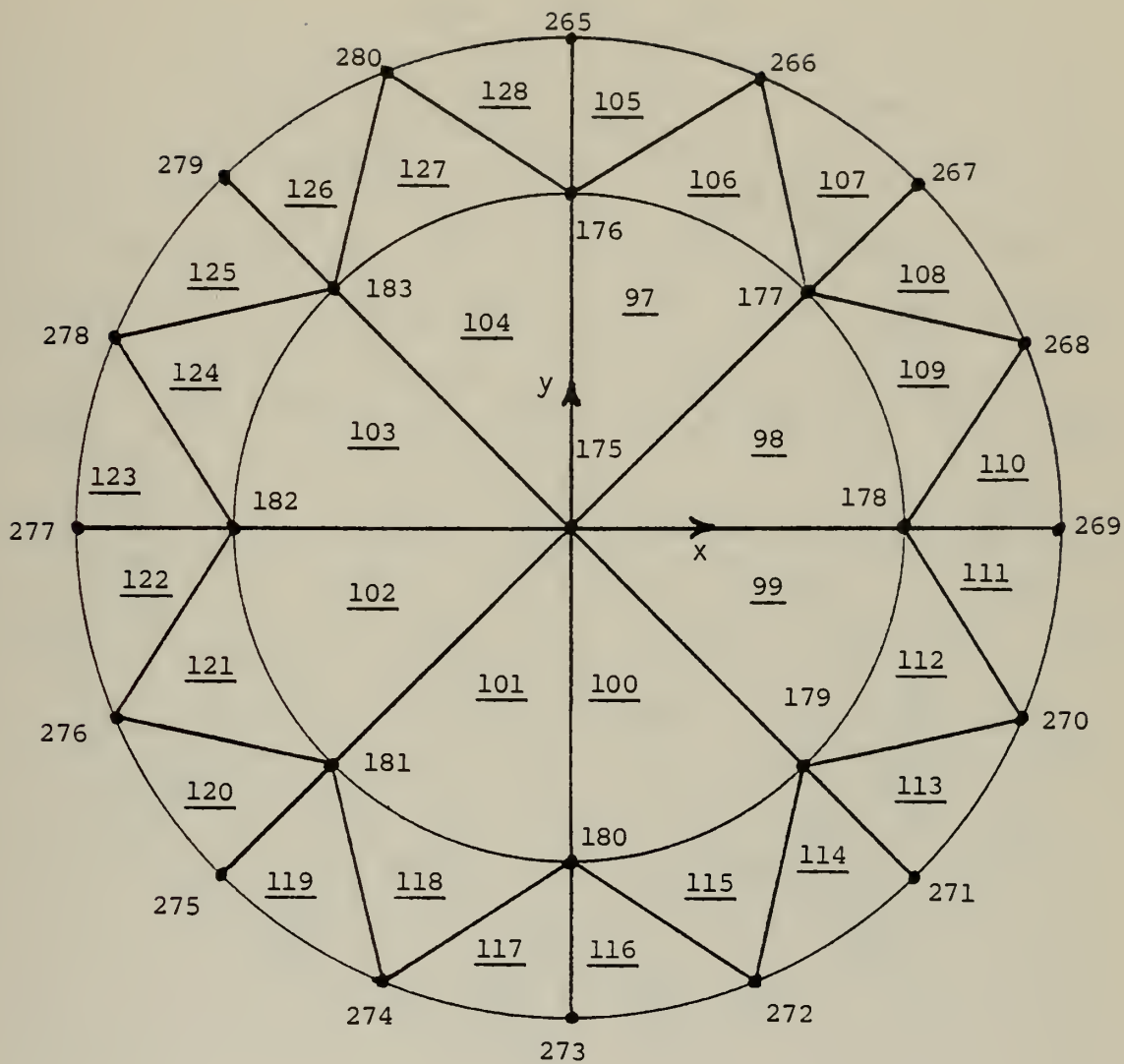
<235, 210, 193, 194, 195, 211, 266, 176, 177, 283, 152, 10, 11, 12, 53>

A second finite element mesh (mesh II) consisting of 192 elements was developed. It contained the same number of elements per layer as mesh I. However, mesh II has six layers. The second and third layers of mesh I were each divided in



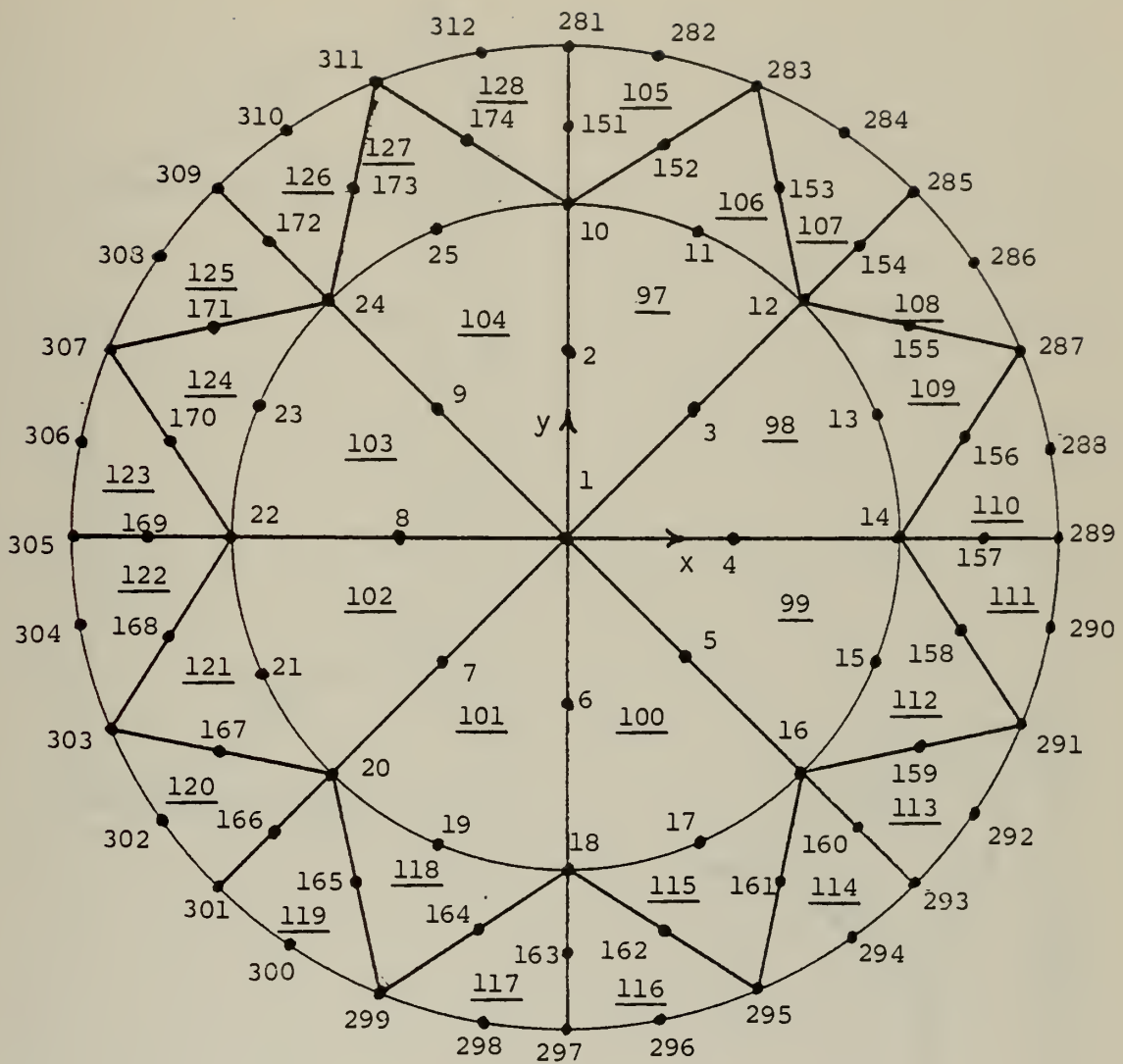
Number - element number

Figure 7. Top nodal plane of the first layer of mesh I,
 $z = 110$ cm



Number - element number

Figure 8. Middle nodal plane of the first layer of mesh I, $z = 95$ cm



Number - element number

Figure 9. Bottom nodal plane of the first layer of mesh I, $z = 80$ cm

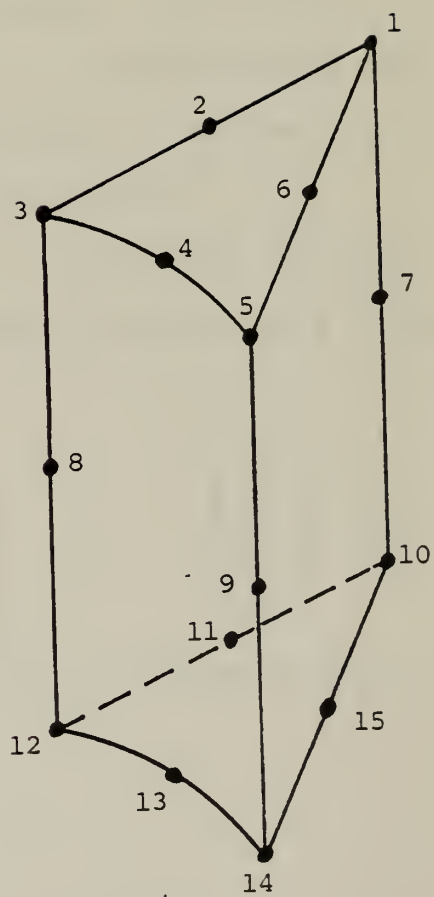
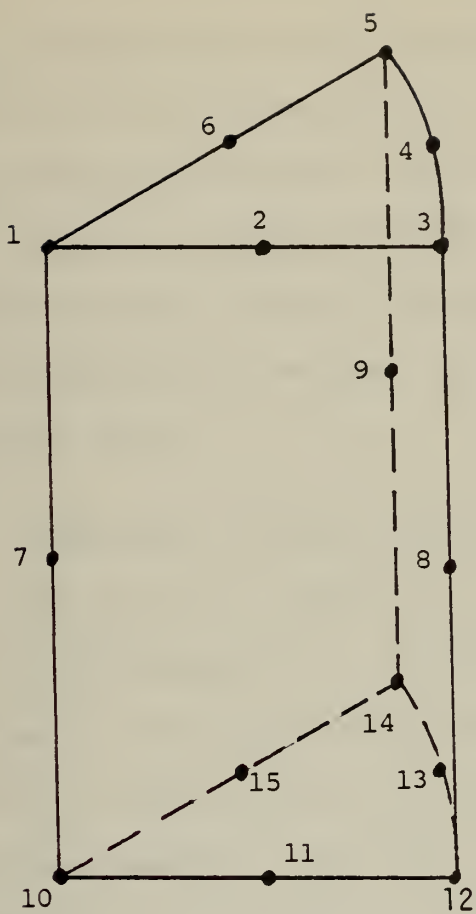


Figure 10. Local nodal numbering of curved elements

half, thus forming the two additional layers of mesh II. The connectivity matrix and the coordinates of each global node for mesh I are given in Appendix A. Appendix B lists the connectivity matrix and nodal coordinates of mesh II. The reader can construct the different layers and elements without much difficulty by using the nodal coordinates and the connectivity matrix. A mesh generator was not utilized in this work.

E. COORDINATE TRANSFORMATION

The use of a quadratic or higher order element permits the transformation or mapping of the straight-sided parent element into an element with curved sides. Distorted or curved elements provide a better fit to curved domains than linear elements, and thus a smaller number of elements is required to represent the structure adequately.

The transformation from cartesian coordinates to curvilinear coordinates can be accomplished by employing a one-to-one correspondence defined by [11]

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = f \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} \quad \text{or} \quad f \begin{pmatrix} L_1 \\ L_2 \\ L_3 \\ \zeta \end{pmatrix} \quad (46)$$

The element shape functions are utilized to achieve this transformation via the relation

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \sum_i N_i x_i \\ \sum_i N_i y_i \\ \sum_i N_i z_i \end{pmatrix}, \quad i=1,2,\dots,n^e \quad (47)$$

where n^e is the number of element nodes, and the element shape functions N_i are in terms of local coordinates. Each set of local coordinates corresponds to only one set of cartesian coordinates.

In performing the transformation, the compatibility requirement must be met. The transformation into the new, curved elements should leave no gaps between adjacent elements. If two adjacent elements are generated from parents in which the element shape functions satisfy continuity requirements, then the curved elements will be contiguous [11]. For the isoparametric element, uniqueness of coordinates ensures compatibility. Continuity is assured when adjacent elements are given the same sets of coordinates at common nodes.

Since the element shape functions are in terms of local coordinates, an element of volume, $dx dy dz$, must be transformed into an element of volume expressed in local coordinates. This is achieved through the use of the Jacobian matrix defined below. Using the chain rule, the relationship between ξ, η, ζ and a corresponding set of cartesian coordinates x, y, z is

$$\begin{Bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \\ \frac{\partial N_i}{\partial \zeta} \end{Bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} & \frac{\partial z}{\partial \zeta} \end{bmatrix} \begin{Bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} \end{Bmatrix} \quad (48)$$

The Jacobian matrix $[J]$ is defined as

$$[J] = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} & \frac{\partial z}{\partial \zeta} \end{bmatrix} \quad (49)$$

From equation (47), the Jacobian matrix becomes

$$[J] = \begin{bmatrix} \sum_i \frac{\partial N_i}{\partial \xi} x_i & \sum_i \frac{\partial N_i}{\partial \xi} y_i & \sum_i \frac{\partial N_i}{\partial \xi} z_i \\ \sum_i \frac{\partial N_i}{\partial \eta} x_i & \sum_i \frac{\partial N_i}{\partial \eta} y_i & \sum_i \frac{\partial N_i}{\partial \eta} z_i \\ \sum_i \frac{\partial N_i}{\partial \zeta} x_i & \sum_i \frac{\partial N_i}{\partial \zeta} y_i & \sum_i \frac{\partial N_i}{\partial \zeta} z_i \end{bmatrix} \quad (50)$$

The determinant of the Jacobian is used to transform the volume of element in cartesian coordinates to local coordinates. For a volume of element [11],

$$dx dy dz = \det [J] d\xi d\eta d\zeta \quad (51)$$

Note that the determinant of the Jacobian is a variable for elements of curved geometry. Only in the case of straight-sided elements is the determinant of the Jacobian a constant.

In the plane of the triangle, the area coordinates (L_1, L_2, L_3) number one more than the cartesian coordinates (x, y). Thus, L_3 is defined as a dependent variable. This establishes the origin of the $\xi\eta$ coordinate system at corner point 3, as

illustrated in figure 11. Recall that the $\xi\eta$ axes need not be orthogonal. As such

$$\xi = L_1 \quad (52a)$$

$$\eta = L_2 \quad (52b)$$

Using equation (42),

$$L_3 = 1 - \xi - \eta \quad (52c)$$

Applying the chain rule yields

$$\frac{\partial N_i}{\partial \xi} = \frac{\partial N_i}{\partial L_1} \frac{\partial L_1}{\partial \xi} + \frac{\partial N_i}{\partial L_2} \frac{\partial L_2}{\partial \xi} + \frac{\partial N_i}{\partial L_3} \frac{\partial L_3}{\partial \xi} \quad (53a)$$

$$\frac{\partial N_i}{\partial \eta} = \frac{\partial N_i}{\partial L_1} \frac{\partial L_1}{\partial \eta} + \frac{\partial N_i}{\partial L_2} \frac{\partial L_2}{\partial \eta} + \frac{\partial N_i}{\partial L_3} \frac{\partial L_3}{\partial \eta} \quad (53b)$$

Using equations (52a), (52b), and (52c) in equations (53a) and (53b) gives

$$\frac{\partial N_i}{\partial \xi} = \frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3} \quad (54a)$$

$$\frac{\partial N_i}{\partial \eta} = \frac{\partial N_i}{\partial L_2} - \frac{\partial N_i}{\partial L_3} \quad (54b)$$

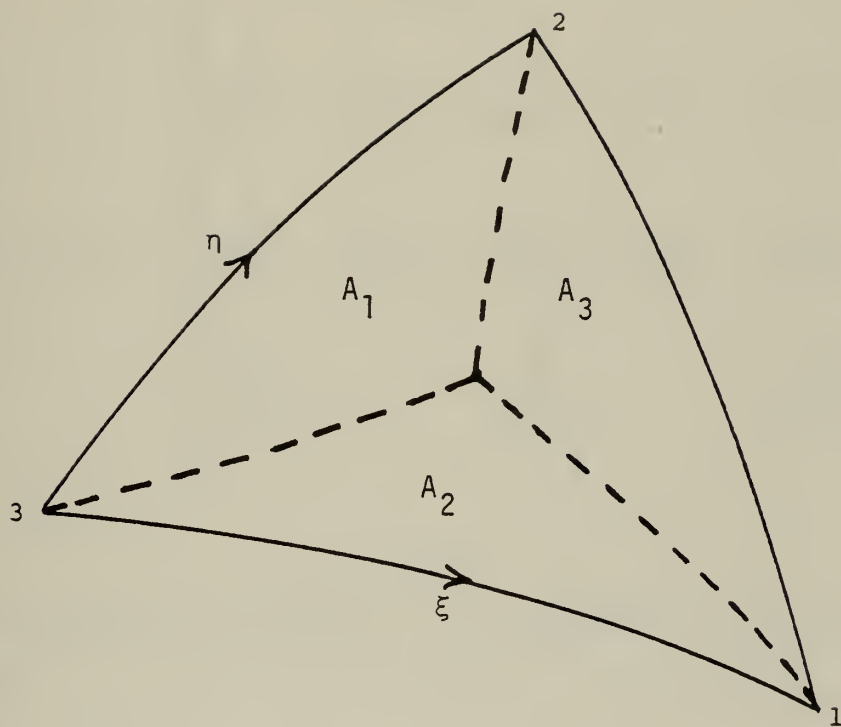


Figure 11. $\eta\xi$ coordinates in a triangle

Now the Jacobian matrix can be evaluated using the shape functions $N_i = N_i(L_1, L_2, L_3, \zeta)$. Inserting equations (54a) and (54b) in equation (50) produces

$$[J(L_1, L_2, L_3, \zeta)] = \begin{bmatrix} \sum_i \left(\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3} \right) x_i & \sum_i \left(\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3} \right) y_i & \sum_i \left(\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3} \right) z_i \\ \sum_i \left(\frac{\partial N_i}{\partial L_2} - \frac{\partial N_i}{\partial L_3} \right) x_i & \sum_i \left(\frac{\partial N_i}{\partial L_2} - \frac{\partial N_i}{\partial L_3} \right) y_i & \sum_i \left(\frac{\partial N_i}{\partial L_2} - \frac{\partial N_i}{\partial L_3} \right) z_i \\ \sum_i \left(\frac{\partial N_i}{\partial \zeta} \right) x_i & \sum_i \left(\frac{\partial N_i}{\partial \zeta} \right) y_i & \sum_i \left(\frac{\partial N_i}{\partial \zeta} \right) z_i \end{bmatrix} \quad (55)$$

To summarize, suppose it is required to transform the integral

$$I = \int_{Vol} F'(L_1, L_2, L_3, \zeta) \, dx dy dz \quad (56)$$

to an integral entirely in terms of local coordinates. The determinant of equation (55) is then utilized to give

$$I = \int_{-1}^1 \int_0^1 \int_0^{1-L_1} F'(L_1, L_2, L_3, \zeta) \det[J(L_1, L_2, L_3, \zeta)] dL_1 dL_2 d\zeta \quad (57)$$

Equation (56) is now in a form suitable for numerical integration.

The development of coordinate transformation to this point has been under the general assumption that all the sides of the element are curved. In this work, the only

curved side is in the plane of the triangle. The sides of the element along the prism axis are straight. As such, a modified Jacobian matrix can be employed, thereby reducing the number of calculations to be performed. This modified Jacobian is the 2x2 matrix defined by

$$[J^*] = \begin{bmatrix} \sum_i \left(\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3} \right) x_i & \sum_i \left(\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3} \right) y_i \\ \sum_i \left(\frac{\partial N_i}{\partial L_2} - \frac{\partial N_i}{\partial L_3} \right) x_i & \sum_i \left(\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3} \right) y_i \end{bmatrix} \quad (58)$$

Along the prism axis or ζ direction, it can be shown that

$$dz = \frac{h}{2} d\zeta \quad (59)$$

where h = height of the element. The volume relationship is then given by

$$dxdydz = \frac{h}{2} \det[J^*] dL_1 dL_2 d\zeta \quad (60)$$

The integral of equation (56) then assumes the form of

$$I = \frac{h}{2} \int_{-1}^1 \int_0^1 \int_0^{1-L_1} F'(L_1, L_2, L_3, \zeta) \det[J^*(L_1, L_2, L_3, \zeta)] dL_1 dL_2 d\zeta \quad (61)$$

Equation (61) is the basis of numerical integration applied in this work.

F. CONSTRUCTION OF ELEMENT MATRICES

The system matrix operators can be constructed through the use of the global basis functions G_j or through the

application of the element shape functions. Although the global basis functions were used to demonstrate the method of Galerkin, the construction of the system matrices in this work was achieved through element considerations. The solution of the unknown variable ψ within the element domain was approximated by

$$\tilde{\psi}^e = \sum_{i=1} N_i \psi_i^e(t) , \quad i=1,2,\dots,n^e \quad (62)$$

where n^e is the number of element nodal points, N_i are the element shape functions, and $\psi_i^e(t)$ are the time-dependent nodal magnitudes of $\tilde{\psi}^e$. The element contribution to the system matrix operators is defined by Galerkin's orthogonality condition expressed by

$$\int_{Vol} N_j R^e dVol = 0 , \quad j=1,2,\dots,n^e \quad (63)$$

where R^e is defined by replacing $\tilde{\psi}$ with $\tilde{\psi}^e$ in equations (37) and (38), and the integration is over the element volume. Using equation (62), the element contribution is portrayed for the core by

$$\begin{aligned} & \left[\int_{Vol} N_j N_i dVol \right] \{ \dot{\psi}_i^e(t) \} - \left[\int_{Vol} N_j \nabla^2 N_i dVol \right] \{ (c1 \cdot \psi^e)_i \} + \left[\int_{Vol} N_j N_i dVol \right] \{ (c2 \cdot \psi^e)_i \} \\ & + \left[\int_{Vol} N_j N_i \ln \left(1 + \frac{K}{\gamma T_0} \sum_k N_k \psi_k^e(t) \right) dVol \right] \{ (c4 \cdot \psi^e)_i \} \\ & + \left[\int_{Vol} N_j N_i dVol \right] \left\{ \int_0^t e^{-\bar{\lambda}(t-t')} (c5 \cdot \psi(t'))_i dt' \right\} = 0 \end{aligned} \quad (64)$$

where $i, j, k = 1, 2, \dots, 15$, and c_1, c_2 , etc., are constants at node i . The bracketed expressions represent square matrices of size 15×15 , and the braced expressions represent column vectors of size 15×1 . For the reflector, the last two terms of equation (64) are zero. Before any operation can be performed on equation (64), the nonlinear terms (last two terms) must be "linearized" and the term with the ∇^2 operator must be integrated by parts.

The nonlinear feedback term, $\lambda_n [1 + \frac{K}{\gamma T_0} \sum_{k=1}^{15} N_k \psi_k^e(t)]$, was linearized by using predicted values of the unknown function at time t . These predicted values come from the time integration scheme by Franke [7]. Basically, the integration scheme utilizes a predictor-corrector method which predicts values of the unknown function at the next time by using the derivatives of the function. Adopting the predicted values ψ_k^P enabled integration over space. The term in equation (64) involving the nonlinear feedback can be written as

$$\left[\int_{Vol} N_j N_i \lambda_n \left(1 + \frac{K}{\gamma T_0} (N_1 \psi_1^P + N_2 \psi_2^P + \dots + N_{15} \psi_{15}^P) \right) dVol \right] \{ (c_4 \cdot \psi^e)_i \}$$

The last term of equation (64) describes the delayed neutron contribution. In general,

$$\begin{aligned} e^{-\bar{\lambda} t_n} \int_0^{t_n} e^{\bar{\lambda} t'} \psi_i^e(t') dt' &= e^{-\bar{\lambda} (t_n - t_{n-1})} \left[e^{-\bar{\lambda} t_{n-1}} \int_0^{t_{n-1}} e^{\bar{\lambda} t'} \psi_i^e(t') dt' \right] \\ &+ e^{-\bar{\lambda} t_n} \int_{t_{n-1}}^{t_n} e^{\bar{\lambda} t'} \psi_i^e(t') dt' \end{aligned} \quad (65)$$

where n is number of time steps, t_n is the current time,

and t_{n-1} is the previous time. To approximate the integrals in equation (65), the predicted values ψ_i^P were employed in a simple trapezoidal rule. The trapezoidal rule was believed to be sufficient since very small time steps were utilized in this work. For example, at time t_2 ,

$$(SUM_i) = (\text{previous } SUM_i) e^{-\bar{\lambda}H} + \frac{H}{2}(e^{\bar{\lambda}H} \psi_i^e(t_1) + \psi_i^P)$$

where

$$H = \text{current time step} = t_2 - t_1$$

$$(SUM_i) = e^{-\bar{\lambda}t_2} \int_0^{t_2} e^{\bar{\lambda}t'} \psi_i^e(t') dt'$$

$$(\text{previous } SUM_i) = e^{-\bar{\lambda}t_1} \int_0^{t_1} e^{\bar{\lambda}t'} \psi_i^e(t') dt'$$

The previous sum is formed by accumulating the trapezoidal integration of each time step. In general then, for time t_n

$$(SUM_i) = (\text{previous } SUM_i) e^{-\bar{\lambda}H} + \frac{H}{2}(e^{\bar{\lambda}H} \psi_i^e(t_{n-1}) + \psi_i^P) \quad (66)$$

The delayed neutron term in equation (64) can be expressed as

$$\left[\int_{Vol} N_j N_i dVol \right] \{ (c5 \cdot SUM)_i \}$$

In order to bring equation (64) to final form, the ∇^2 operator was integrated by parts. Since the flux at the surface of the reactor is zero, integration by parts yields

$$\int_{Vol} N_j \nabla^2 N_i dx dy dz = - \int_{Vol} \left(\frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} + \frac{\partial N_i}{\partial z} \frac{\partial N_j}{\partial z} \right) dx dy dz \quad (67)$$

In vector notation, equation (67) is expressed as

$$\int_{Vol} N_j \nabla^2 N_i dx dy dz = - \int_{Vol} \left\langle \frac{\partial N_i}{\partial x}, \frac{\partial N_i}{\partial y}, \frac{\partial N_i}{\partial z} \right\rangle \begin{Bmatrix} \frac{\partial N_j}{\partial x} \\ \frac{\partial N_j}{\partial y} \\ \frac{\partial N_j}{\partial z} \end{Bmatrix} dx dy dz \quad (68)$$

Using the chain rule produces

$$\begin{Bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} \end{Bmatrix} = \begin{bmatrix} \frac{\partial L_1}{\partial x}, \frac{\partial L_2}{\partial x}, 0 \\ \frac{\partial L_1}{\partial y}, \frac{\partial L_2}{\partial y}, 0 \\ 0, 0, \frac{\partial \zeta}{\partial z} \end{bmatrix} \begin{Bmatrix} \left(\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3} \right) \\ \left(\frac{\partial N_i}{\partial L_2} - \frac{\partial N_i}{\partial L_3} \right) \\ \left(\frac{\partial N_i}{\partial \zeta} \right) \end{Bmatrix} \quad (69)$$

Letting the 3x3 matrix above be $[B']$, the ∇^2 term becomes

$$\begin{aligned} \int_{Vol} N_j \nabla^2 N_i dx dy dz = & - \int_{Vol} \left\langle \left(\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3} \right), \left(\frac{\partial N_i}{\partial L_2} - \frac{\partial N_i}{\partial L_3} \right), \frac{\partial N_i}{\partial \zeta} \right\rangle [B']^T [B'] \begin{Bmatrix} \left(\frac{\partial N_j}{\partial L_1} - \frac{\partial N_j}{\partial L_3} \right) \\ \left(\frac{\partial N_j}{\partial L_2} - \frac{\partial N_j}{\partial L_3} \right) \\ \frac{\partial N_j}{\partial \zeta} \end{Bmatrix} dx dy dz \end{aligned} \quad (70)$$

where $[B']^T$ is the transpose of $[B']$. By applying the chain rule in equation (47) and using equation (59), it can be shown that for this work

$$[B'] = \begin{bmatrix} \frac{1}{(\frac{\partial N_1}{\partial L_1} x_1 + \dots + \frac{\partial N_{15}}{\partial L_1} x_{15})}, \frac{1}{(\frac{\partial N_1}{\partial L_2} x_1 + \dots + \frac{\partial N_{15}}{\partial L_2} x_{15})}, 0 \\ \frac{1}{(\frac{\partial N_1}{\partial L_1} y_1 + \dots + \frac{\partial N_{15}}{\partial L_1} y_{15})}, \frac{1}{(\frac{\partial N_1}{\partial L_2} y_1 + \dots + \frac{\partial N_{15}}{\partial L_2} y_{15})}, 0 \\ 0, 0, \frac{2}{h} \end{bmatrix} \quad (71)$$

$[B']^T$ can be derived from equation (71).

Note from equation (64) that there are three basic element matrices which are defined as follows after applying equation (60):

$$[G_{ji}] = \frac{h}{2} \int_{-1}^1 \int_0^1 \int_0^{1-L_1} N_j N_i \det[J^*] dL_1 dL_2 d\zeta \quad (72)$$

$$[GG_{ji}] = \frac{h}{2} \int_{-1}^1 \int_0^1 \int_0^{1-L_1} \left\langle \frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3}, \left(\frac{\partial N_i}{\partial L_2} - \frac{\partial N_i}{\partial L_3} \right), \left(\frac{\partial N_i}{\partial \zeta} \right) \right\rangle [B']^T$$

$$[B'] \left\{ \begin{array}{c} \left(\frac{\partial N_j}{\partial L_1} - \frac{\partial N_j}{\partial L_3} \right) \\ \left(\frac{\partial N_j}{\partial L_2} - \frac{\partial N_j}{\partial L_3} \right) \\ \frac{\partial N_j}{\partial \zeta} \end{array} \right\} \det[J^*] dL_1 dL_2 d\zeta \quad (73)$$

$$[GGG_{ji}] = \frac{h}{2} \int_{-1}^1 \int_0^1 \int_0^{1-L_1} N_j N_i \ln \left(1 + \frac{K}{\gamma T_0} (N_1 \psi_1^P + \dots + N_{15} \psi_{15}^P) \right) \det[J^*] dL_1 dL_2 d\zeta \quad (74)$$

Element matrices $[G_{ji}]$ and $[GG_{ji}]$ are independent of time. However, $[GGG_{ji}]$ is time dependent due to the utilization of the predicted values ψ_i^p which changes with time. In terms of these three basic element matrices, equation (64) is

$$[G_{ji}]\{\dot{\psi}_i^e\} + [GG_{ji}]\{c1 \cdot \psi_i^e\} + [G_{ji}]\{(c2 \cdot \psi_i^e)\} \\ + [GGG_{ji}]\{(c4 \cdot \psi_i^e)\} + [G_{ji}]\{(c5 \cdot \text{SUM})_i\} = 0 \quad (75)$$

The last two terms of equation (75) are zero for the reflector.

G. CONSTRUCTION OF THE SYSTEM MATRICES

The 15x15 coefficient element matrices were calculated according to equations (72), (73), and (74); and the results were collected element by element into the corresponding system coefficient matrices. The system coefficient matrix $[BIGG]$ is developed from $[G_{ji}]$, $[BIGGG]$ from $[GG_{ji}]$, and $[BIGH]$ from $[GGG_{ji}]$. $[BIGG]$ and $[BIGGG]$ are independent of time and can be constructed once and for all from geometry considerations. $[BIGH]$ is dependent on both geometry and time due to the time dependence of the predicted flux utilized in the feedback term. Thus, $[BIGH]$ is recalculated at each time increment.

Non-zero contributions to a global nodal point I come only from adjacent elements sharing that same nodal point I. Thus, the system matrices are sparse and banded. The process of assembling contributions from element matrices

requires the identification of a local nodal point ($i=1,2,\dots,15$) with a global nodal point ($I=1,2,\dots,\text{NUMNP}$, where NUMNP is the total number of system nodes). This correspondence between element and global nodes is accomplished via the connectivity matrix.

The formal treatment of the field equations in terms of the system coefficient matrices is described by the equation

$$[\text{BIGG}] \{\dot{\psi}_I\} + [\text{BIGGG}] \{(c1 \cdot \psi)_I\} + [\text{BIGG}] \{(c2 \cdot \psi)_I\} + [\text{BIGH}] \{(c4 \cdot \psi)_I\} + [\text{BIGG}] \{(c5 \cdot \text{SUM})_I\} = 0 \quad (76)$$

where the system matrices are $\text{NUMNP} \times \text{NUMNP}$ and the column vectors are of length $\text{NUMNP} \times 1$. However, the direct application of equation (76) requires a large amount of computer storage. To take advantage of the sparsity of the system matrices, an optimum compacting scheme described by Ref. 8 was employed.

The concept behind OCS is simply to store only the non-zero terms of a coefficient matrix. OCS requires two integer arrays, say JB and NAME , and a vector of non-zero coefficients of the square system matrix. For purposes of illustration, the square system matrix is called B , and the vector of non-zero coefficients of B is called BB . The i^{th} integer entry in the $\text{NUMNP} \times 1$ JB vector is the number q_i . This number is defined by

$$q_i = 1 + \sum_{j=1}^{i=1} p_j, \quad i=1,2,\dots,\text{NUMNP} \quad (77)$$

where p_j is the number of terms in the i^{th} equation. In

other words, p_j is the number of nodes that the i^{th} node "sees". JB is therefore a pointer vector of length NUMNP+1 whose i^{th} term locates the initial position in the BB vector of the contributing coefficients to the i^{th} equation. The NAME vector of length Mx1, where $M = \sum_{i=1}^{\text{NUMNP}} p_i$, consists of NUMNP successive vector blocks of variable length p_i , $i=1,2,\dots,\text{NUMNP}$. The p_i integer numbers in the i^{th} block of NAME list the p_i contributors to the i^{th} equation. The Mx1 BB vector contains the real non-zero coefficients of the NUMNPxNUMNP B matrix, arranged in the same contiguous block arrangement as the NAME vector. The j^{th} term in the i^{th} block or $\text{BB}(\text{JB}(I) + J - 1)$ is $B(I,K)$, where $K = \text{NAME}(\text{JB}(I) + J - 1)$. To illustrate, consider the grid shown in figure 12. The two array vectors and the coefficient vector matrix of non-zero terms are:

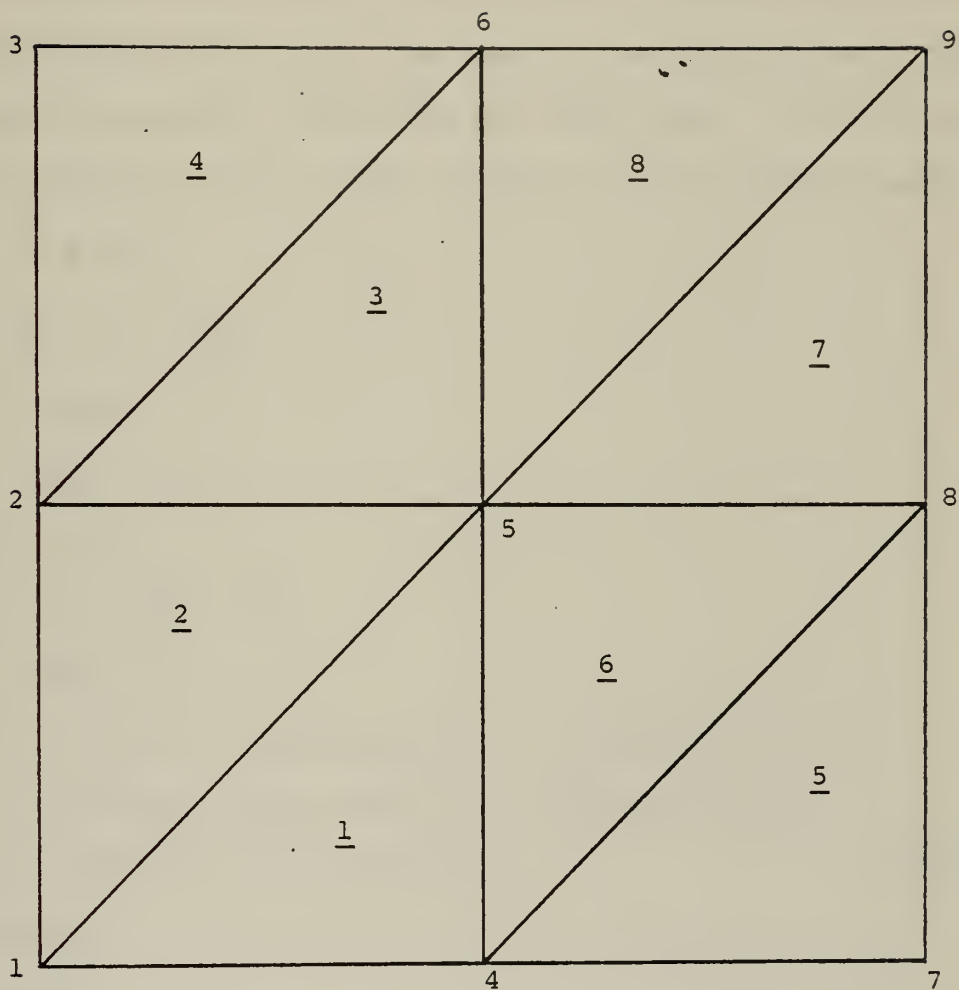
$$\text{JB} = \langle 1, 5, 10, 13, 18, 25, 30, 33, 38, 42 \rangle$$

$$\text{NAME} = \langle 1, 2, 4, 5 | 2, 3, 1, 6, 5 | \text{---} | 9, 8, 6, 5 \rangle$$

$$\text{BB} = \langle B_{11}, B_{12}, B_{14}, B_{15} | B_{22}, B_{23}, B_{21}, B_{26}, B_{25} | \text{---} | B_{99}, B_{98}, B_{96}, B_{95} \rangle$$

In this illustration, NUMNP = 9 and M = 41 [8].

In this work, a judicious method of numbering the system nodal points was adopted to further reduce computer storage requirements. Since the surface boundary nodes of the reactor represent zero neutron fluxes, the contributions of these nodes to interior or non-zero nodes can be discarded. Thus, only the interior nodal points need to be considered. These non-zero nodes were numbered first in the finite element mesh



Number - element number

Figure 12. Sample grid used for illustrating OCS

used so that in the OCS, the number of non-zero nodes (NNZ) replaces NUMNP.

The vectors of non-zero coefficients will be designated BIGG, BIGGG and BIGH since the square coefficient matrices described in equation (76) were not utilized. To illustrate the application of OCS in the system, the following sample program is given:

```
DO 450 I=1, NNZ
JBB = JB(I)
JE = JB(I+1)-1
DY(I) = 0.0
DO 500 J=JBB, JE
LL = NAME(J)
DY(I) = DY(I) + BIGG(J)* $\dot{\psi}$ (LL) + BIGGG(J)*c1(LL)*
 $\psi$ (LL) + BIGG(J)*c2(LL)* $\psi$ (LL) + BIGH(J)*c4(LL)*
 $\psi$ (LL) + BIGG(J)*c5(LL)*SUM(LL) (78)
500 CONTINUE
450 CONTINUE
```

where

LL = nodal point "seen" by node I

JE-JBB = total number of nodal points that node I "sees"

DY(I) = summation of all contributions to node I and
should sum to zero as stated by the field equation

The assumption of a homogeneous reflector was relaxed in the application of equation (78). Interface nodes were assigned properties of the core. Therefore, if LL is a reflector

node not on the core reflector interface, the last two terms of equation (78) are nonexistent.

IV. NUMERICAL INTEGRATION

A. LINE AND AREA INTEGRATION

The solution of the element matrix equations was achieved through numerical integration since an exact closed form solution cannot be established. The volume integration was accomplished by using a line integration in the ζ -direction and an area integration in the plane of the triangle. The line integral is described by the Gaussian quadrature formula [12]

$$\int_{-1}^1 f(\zeta) d\zeta \approx \sum_{k=1}^n H_k f(a_k) \quad (79)$$

where n is the number of Gauss integration points

H_k = weighting coefficients

$f(a_k)$ = the function $f(\zeta)$ evaluated at Gauss point a_k

Table III lists a_k , H_k and n [11].

The area integration was achieved by the equation

$$\int_0^1 \int_0^{1-L_1} f(L_1, L_2, L_3) dL_1 dL_2 \approx \sum_{m=1}^{\bar{m}} w_m f(L_1^m, L_2^m, L_3^m) \quad (80)$$

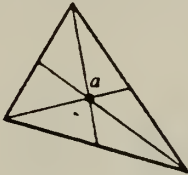
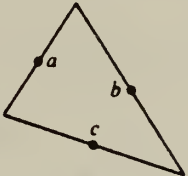
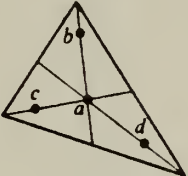
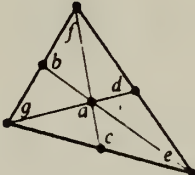
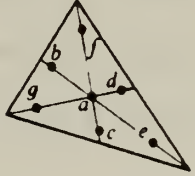
where \bar{m} is the number of area integration points and w_m are the weights. The numerical integration points for the area integration are given in Table IV which was extracted

TABLE III

Abscissae and Weight Coefficients
of the Gaussian Quadrature Formula

$\pm a$			H		
$n = 2$					
0.57735	02691	89626	1.00000	00000	00000
$n = 3$					
0.77459	66692	41483	0.55555	55555	55556
0.00000	00000	00000	0.88888	88888	88889
$n = 4$					
0.86113	63115	94053	0.34785	48451	37454
0.33998	10435	84856	0.65214	51548	62546
$n = 5$					
0.90617	98459	38664	0.23692	68850	56189
0.53846	93101	05683	0.47862	86704	99366
0.00000	00000	00000	0.56888	88888	88889
$n = 6$					
0.93246	95142	03152	0.17132	44923	79170
0.66120	93864	66265	0.36076	15730	48139
0.23861	91860	83197	0.46791	39345	72691
$n = 7$					
0.94910	79123	42759	0.12948	49661	68870
0.74153	11855	99394	0.27970	53914	89277
0.40584	51513	77397	0.38183	00505	05119
0.00000	00000	00000	0.41795	91836	73469
$n = 8$					
0.96028	98564	97536	0.10122	85362	90376
0.79666	64774	13627	0.22238	10344	53374
0.52553	24099	16329	0.31370	66458	77887
0.18343	46424	95650	0.36268	37833	78362
$n = 9$					
0.96816	02395	07626	0.08127	43883	61574
0.83603	11073	26636	0.18064	81606	54857
0.61337	14327	00590	0.26061	06964	02935
0.32425	34234	03809	0.31234	70770	40003
0.00000	00000	00000	0.33023	93550	01260
$n = 10$					
0.97390	65285	17172	0.06667	13443	08688
0.86506	33666	88985	0.14945	13491	50581
0.67940	95682	99024	0.21908	63625	15982
0.43339	53941	29247	0.26926	67193	09996
0.14887	43389	81631	0.29552	42247	14753

TABLE IV
Numerical Formulas for Triangles

Order	Fig.	Error	Points	Triangular Co-ordinates	Weights $2W_i$
Linear		$R = O(h^2)$	a	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$	1
Quadratic		$R = O(h^3)$	a b c	$\frac{1}{2}, \frac{1}{2}, 0$ $0, \frac{1}{2}, \frac{1}{2}$ $\frac{1}{2}, 0, \frac{1}{2}$	$\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$
Cubic		$R = O(h^4)$	a b c d	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$ $\frac{2}{3}, \frac{1}{3}, \frac{1}{3}$ $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$	$-\frac{27}{248}$ $\frac{27}{248}$
Cubic		$R = O(h^4)$	a b c d e f g	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ $\frac{1}{2}, \frac{1}{2}, 0$ $0, \frac{1}{2}, \frac{1}{2}$ $\frac{1}{2}, 0, \frac{1}{2}$ $1, 0, 0$ $0, 1, 0$ $0, 0, 1$	$\frac{27}{80}$ $\frac{8}{80}$ $\frac{3}{80}$
Quintic		$R = O(h^6)$	a b c d e f g	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ $\alpha_1, \beta_1, \beta_1$ $\beta_1, \alpha_1, \beta_1$ $\beta_1, \beta_1, \alpha_1$ $\alpha_2, \beta_2, \beta_2$ $\beta_2, \alpha_2, \beta_2$ $\beta_2, \beta_2, \alpha_2$	0.225 0.13239415 0.12593918
with $\alpha_1 = 0.05971587$ $\beta_1 = 0.47014206$ $\alpha_2 = 0.79742699$ $\beta_2 = 0.10128651$					

from Ref. 11. The volume integration of the function $f(L_1, L_2, L_3, \zeta)$ using numerical integration is therefore

$$\int_{-1}^1 \int_0^1 \int_0^{1-L_1} f(L_1, L_2, L_3, \zeta) dL_1 dL_2 d\zeta \approx \sum_{k=1}^{\bar{n}} w_k \sum_{m=1}^{\bar{m}} w_m f(L_1^m, L_2^m, L_3^m, \zeta^k) \quad (81)$$

where $w_k = H_k$. In the manner of equation (81), the element matrices become

$$[G_{ji}] = \frac{h}{2} \sum_{k=1}^{\bar{n}} w_k \sum_{m=1}^{\bar{m}} w_m N_j(L_1^m, L_2^m, L_3^m, \zeta^k) N_i \det[J^*] \quad (82)$$

$$[GG_{ji}] = \frac{h}{2} \sum_{k=1}^{\bar{n}} w_k \sum_{m=1}^{\bar{m}} w_m F(L_1^m, L_2^m, L_3^m, \zeta^k) \det[J^*] \quad (83)$$

$$[GGG_{ji}] = \frac{h}{2} \sum_{k=1}^{\bar{n}} w_k \sum_{m=1}^{\bar{m}} w_m T(L_1^m, L_2^m, L_3^m, \zeta^k) \det[J^*] \quad (84)$$

where F and T are easily derived from equations (73) and (74), respectively. Note that N_i and $\det[J^*]$ are also evaluated at each integration point. They are not shown as such merely for the sake of convenience in writing the equations.

B. NUMBER OF INTEGRATION POINTS

It is difficult to estimate the number of integration points required for good accuracy due to the complexity of the functions involved. The basic rule that the best number of integration points is found by trial and experience was adopted. For the $[G_{ji}]$ element matrix, the numbers involved

can be approximated. This was done by letting the determinant of the Jacobian equal to twice the area of the curved triangle (checking $\det[J^*]$ at each integration point showed that this assumption was not too unreasonable). With $\det[J^*]$ outside the integration process, $[G_{ji}]$ can be solved in closed form by integrating out ζ from -1 to +1 and then applying the closed form equation [12]

$$\int_{\text{Area}} L_1^{m_1} L_2^{m_2} L_3^{m_3} d(\text{Area}) = 2A \frac{m_1! m_2! m_3!}{(m_1 + m_2 + m_3 + 2)!} \quad (85)$$

where m_1, m_2, m_3 are positive integer exponents and A is the area of the triangle.

Five test points within the 15x15 element matrix were selected, as listed in Table V. The values obtained from the application of equation (85) are also given in Table V. Three sets of area integration points, each with a different number of ζ Gauss points, were used. These three sets of area integration points, given in Table IV, are:

1. cubic order (4 points)
2. cubic order (7 points)
3. quintic order (7 points)

Using each of the three integration points above with different ζ Gauss points in equation (82) produced the results obtained in Table V. From these results, the quintic order area integration points were selected for the element matrix $[G_{ji}]$ with three ζ Gauss points in the prism axes. This set of integration points was also used for $[GGG_{ji}]$.

TABLE V.

Selection of Integration Points for $[G_{ji}]$

ζ points	Area points	$G(1,1)$	$G(2,2)$	$G(1,9)$	$G(6,9)$	$G(9,9)$
13	cubic (4 pts)	1415	5278	-2756	5072	8536
5	"	1817	5278	-3170	5072	10240
7	"	1817	5278	-3170	5072	10240
3	cubic (7 pts)	3248	8247	-3114	4960	10243
5	"	3249	8247	-3114	4960	10240
7	"	3249	8247	-3114	4960	10240
3	quintic (7 pts)	2464	6600	-3144	5020	10240
5	"	2464	6600	-3144	5020	10240
7	"	2464	6600	-3144	5020	10240
Approximated values		2500	6700	-3142	5026	10053

The numbers for the $[GG_{ji}]$ element matrix could not be approximated. The best that could be done was to obtain an idea of the order of magnitude of this element matrix. Towards this end, a linear triangular element was assumed. Using linear approximation, the order of magnitude was found to be about 10^3 . Using the three different area integration points mentioned above with varying ζ Gauss points in equation (83) yielded the results given in Table VI. Further checks on the $[GG_{ji}]$ element matrix showed that the cubic order with four area integration points yielded the desired order of magnitude. Thus, the fourth order cubic with five ζ Gauss points was employed for the $[GG_{ji}]$ element matrix. A note should be mentioned here in regards to the vast difference in results obtained for $[GG_{ji}]$ using different area integration points. Most likely, it was due to the $[B']$ and $[B']^T$ matrices which required the inversion of $\frac{\partial x}{\partial L_1}$, $\frac{\partial x}{\partial L_2}$, etc. Or perhaps it was caused by the nature of the hybrid element used in this work. In any case, further investigation is warranted in this area.

TABLE VI

Selection of Integration Points for $[GG_{ji}]$

ζ points	Area points	GG(1,9)	GG(3,3)	GG(14,9)	GG(8,8)	GG(15,15)
3	cubic (4 pts)	18.4	170.1	-165	165.9	106.1
5	"	22.0	177.6	-186	195.9	106.1
7	"	22.0	177.6	-186	195.9	106.1
3	cubic (7 pts)	-1.03×10^4	2.11×10^7	1.05×10^7	8.42×10^7	2056
5	"	-1.03×10^4	2.11×10^7	1.05×10^7	8.42×10^7	2056
7	"	-1.03×10^4	2.11×10^7	1.05×10^7	8.41×10^7	2056
3	quintic (7 pts)	-52.6	1.27×10^4	-6.64×10^4	6.84×10^4	1.51×10^5
5	"	-52.6	1.27×10^4	-6.64×10^4	6.84×10^4	1.51×10^5
7	"	-52.6	1.27×10^4	-6.64×10^4	6.84×10^4	1.51×10^5

V. TEST PROBLEMS AND RESULTS

The reactor was subjected to uniform and local perturbations in the form of a ramp input described by

$$\Sigma_f = \Sigma_f^* + \alpha t \quad (86)$$

where α is the change in Σ_f per unit time and Σ_f^* is the critical fission cross-section. Σ_f^* must first be obtained before the perturbations can be applied. This was accomplished by trial and error until a stationary solution was reached. For mesh I, Σ_f^* was found to be 0.0057360 per cm. No attempt was made to find Σ_f^* for mesh II due to time limitations. As such, the test problems outlined below were applied to mesh I. Future work is planned to apply the test problems on mesh II.

The following perturbations were applied:

a) Uniform perturbation of 10 dollar of reactivity per second:

$$\Sigma_f(\bar{r}, t) = \Sigma_f^* + \alpha t, \quad \text{in the core}$$

where $\alpha = 0.005893/\text{cm-sec}$

b) Local perturbation at the core center of 100 dollar of reactivity per second:

$$\Sigma_f(\bar{r}, t) = \Sigma_f^* + \alpha t \delta(\bar{r}_0), \quad \text{in the core}$$

where \bar{r}_0 is (0,0,0) and $\alpha = 0.015407/\text{cm-sec}$

c) Local, off-center perturbation:

$$\Sigma_f(\bar{r},t) = \Sigma_f^* + \alpha_i t \delta(\bar{r}_1) , \quad i = 1,2,3, \text{ in the core}$$

where

$$\bar{r}_1 = (0,60,40)$$

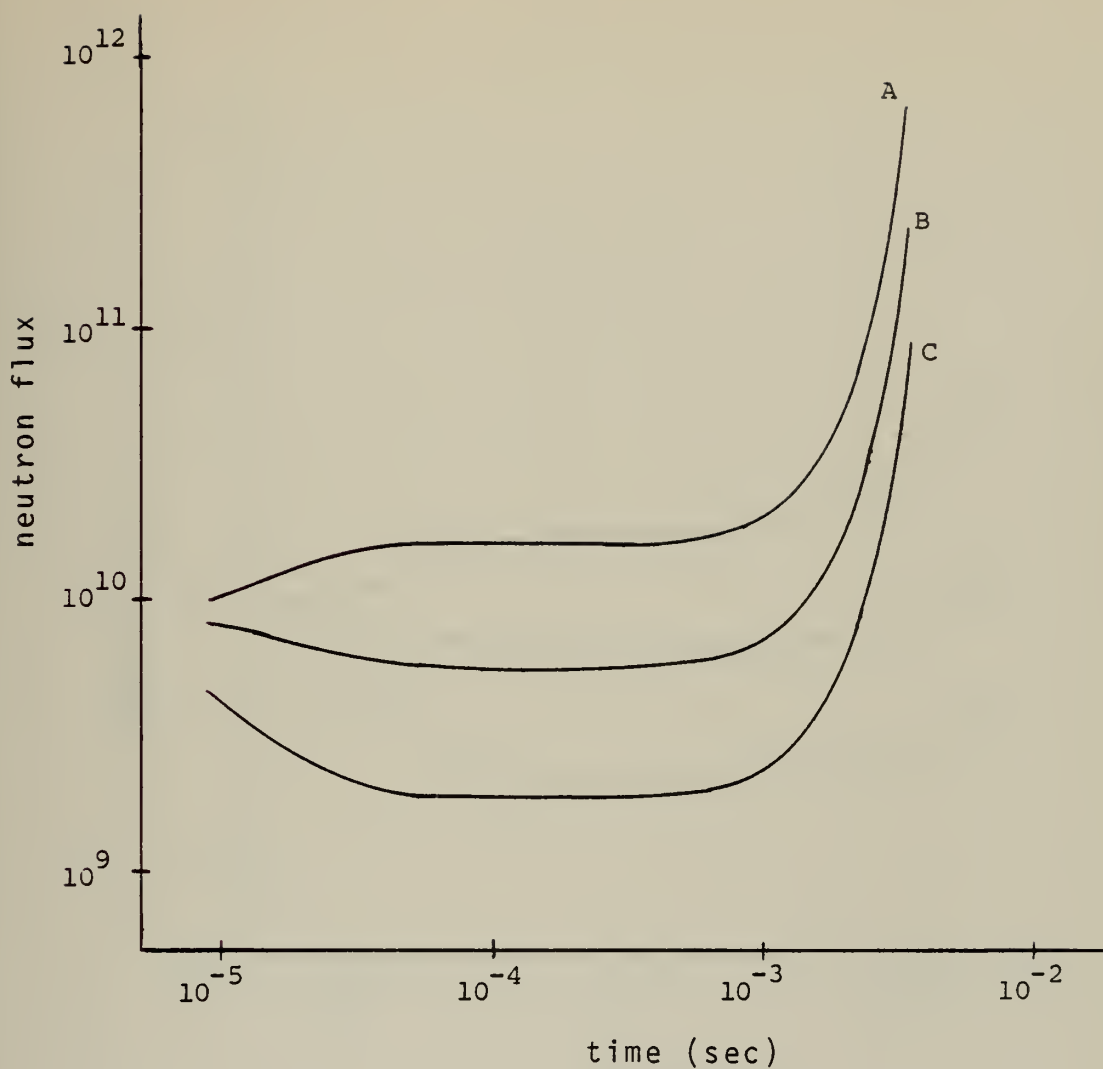
$$\alpha_1 = 0.015407/\text{cm-sec} = 100 \text{ dollar per second}$$

$$\alpha_2 = 0.008123/\text{cm-sec} = 50 \text{ dollar per second}$$

$$\alpha_3 = 0.005894/\text{cm-sec} = 10 \text{ dollar per second}$$

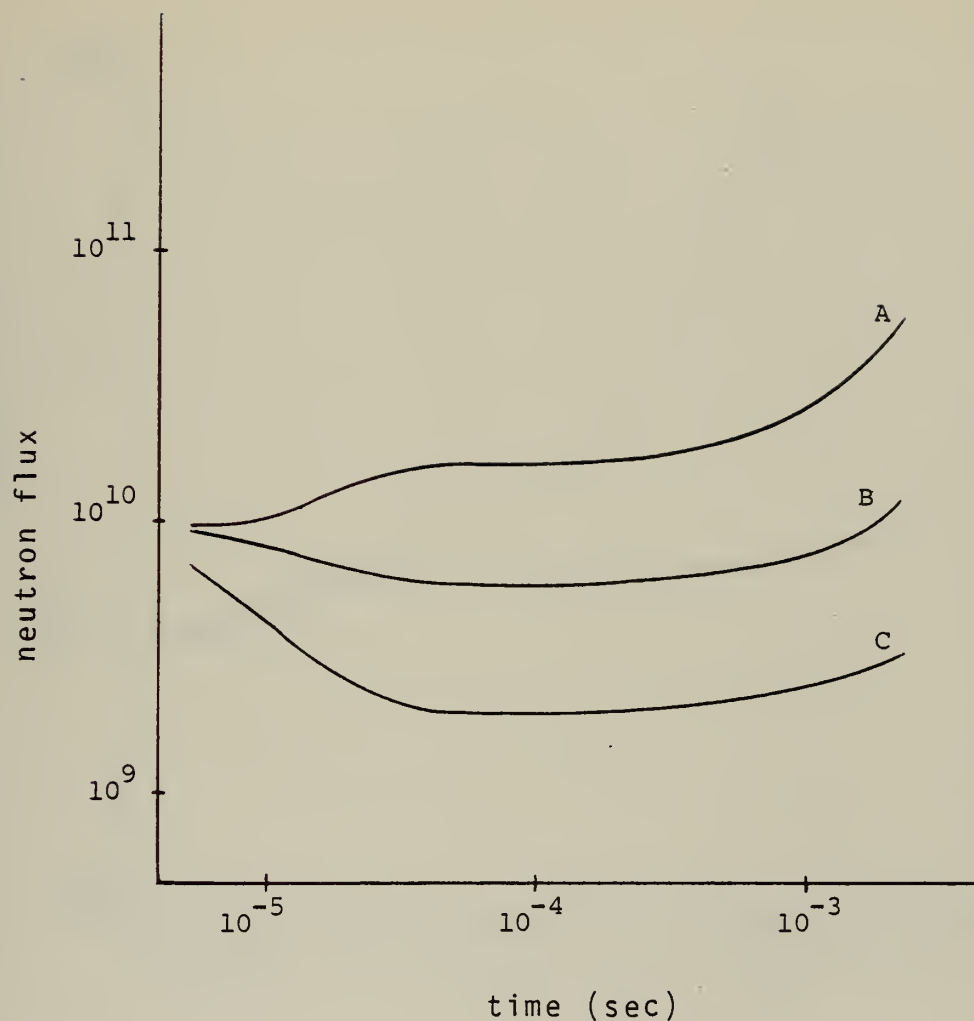
Three test points, (0,0,0), (60,0,0), and (-60,0,80), were selected to trace the neutron time history. For cases a) and b), the neutron flux was plotted at each test point during transience. This is shown in figures 13 and 14. Case c) involved three ramp inputs and were conducted for both the linear and nonlinear reactor equations. The linear and nonlinear responses were compared at each test point for each ramp input and are illustrated in figures 15 through 23. The radial and axial flux distributions at time $t = 0.0123$ second were plotted for the steady state and the 100 dollar perturbation case. These are embodied in figures 24 and 25. Finally, a neutron flux early time history between mesh I and mesh II was plotted to check the effect of using a finer element mesh. The result is portrayed in figure 26.

Figures 13 and 14 revealed a clear space dependence of the neutron flux during transience as expected. Figures 15 through 23 demonstrated the effects of temperature feedback



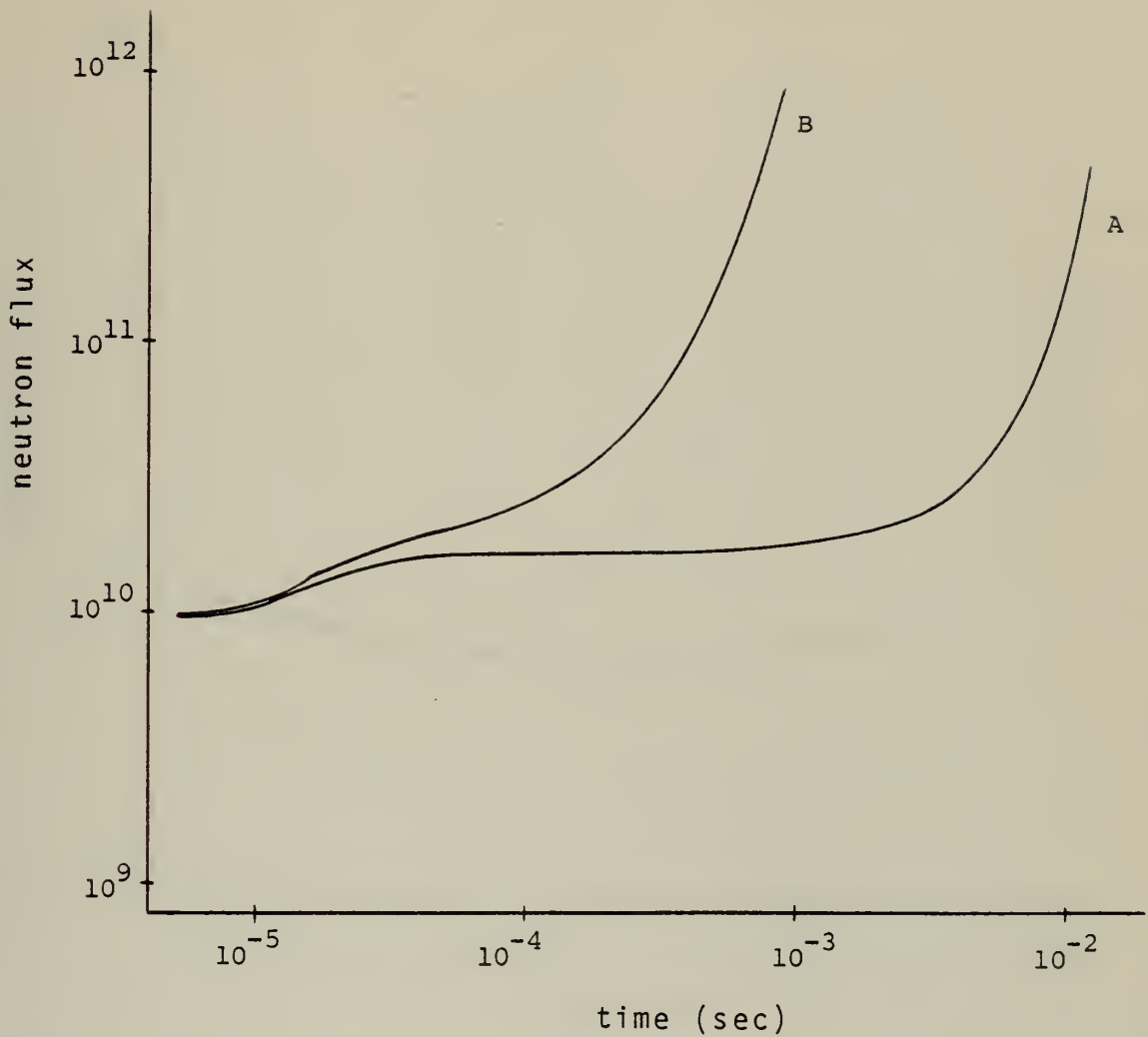
A - neutron flux at test point (0,0,0)
 B - neutron flux at test point (60,0,0)
 C - neutron flux at test point (-60,0,80)

Figure 13. Neutron flux transient history at three test points with a uniform perturbation of 10 dollar of reactivity per second.



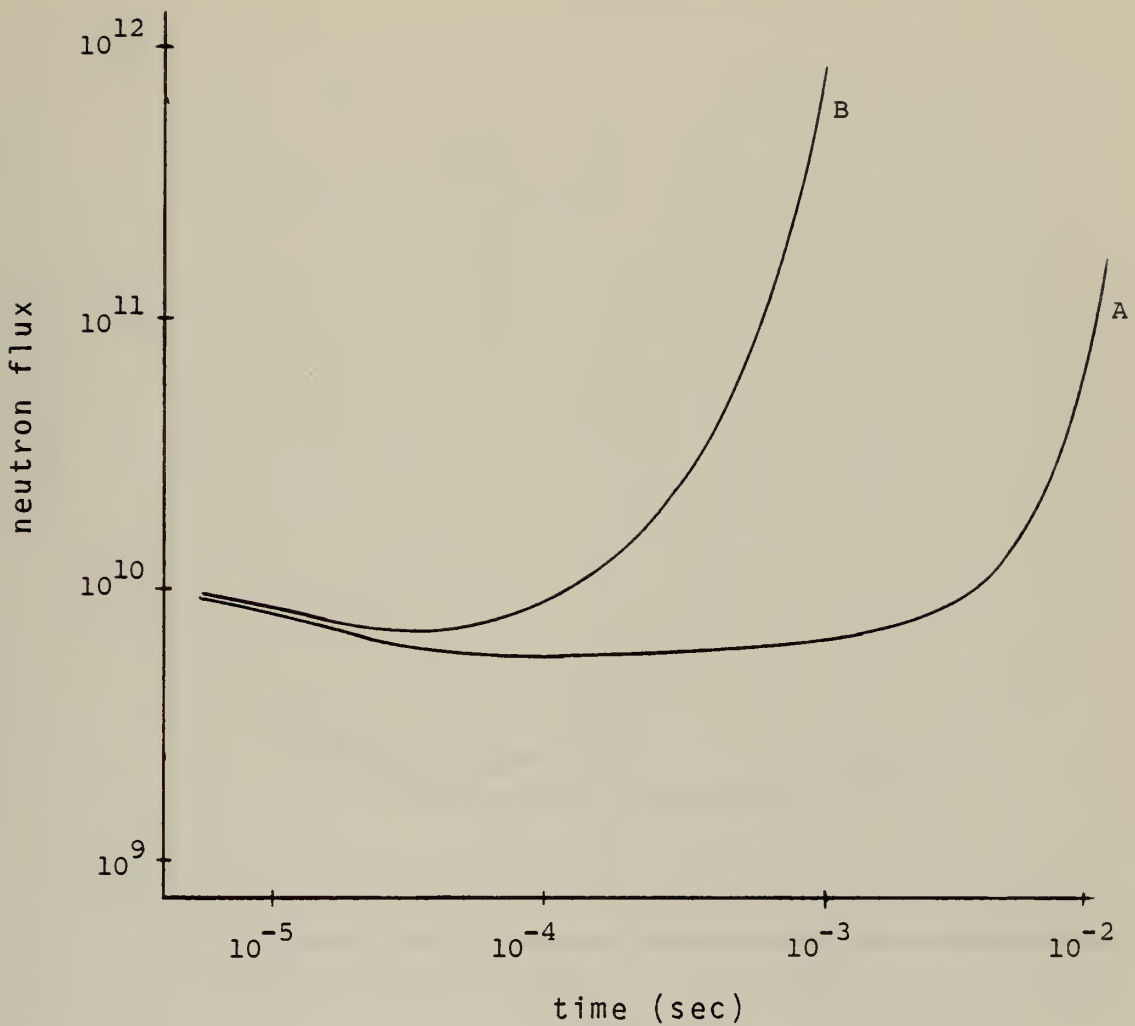
A - neutron flux at test point (0,0,0)
 B - neutron flux at test point (60,0,0)
 C - neutron flux at test point (-60,0,80)

Figure 14. Neutron flux transient history at various test points for a local central perturbation of 100 dollar/sec of reactivity.



A - nonlinear neutron response
B - linear neutron response

Figure 15. Linear and nonlinear fluxes at (0,0,0) due to a local perturbation of 100 dollar/sec of reactivity at (0,60,40).



A - nonlinear neutron response
B - linear neutron response

Figure 16. Linear and nonlinear fluxes at (60,0,0) due to a local perturbation of 100 dollar/sec at (0,60,40).

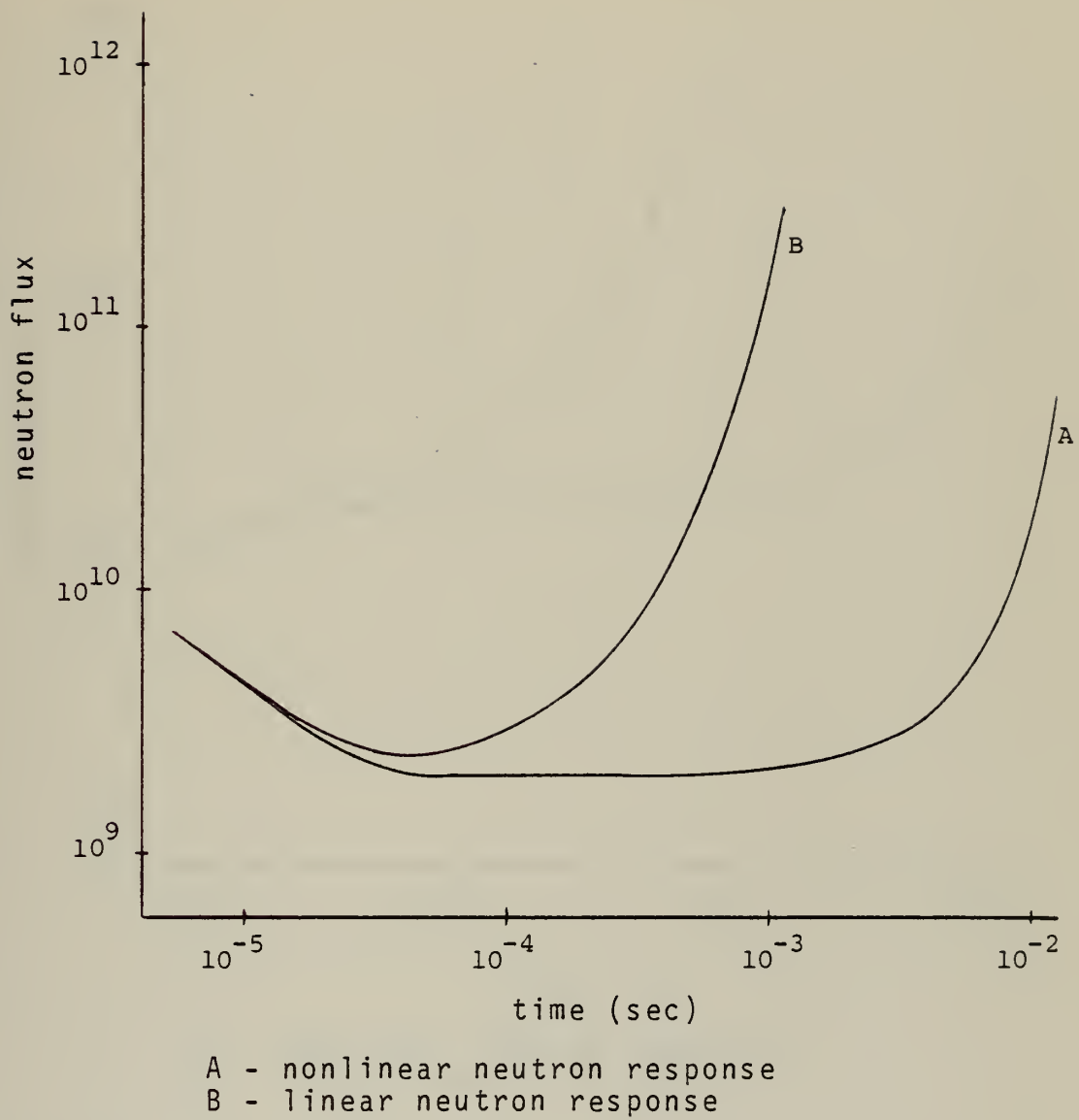


Figure 17. Linear and nonlinear fluxes at $(-60, 0, 80)$ due to a local perturbation of 100 dollar/sec at $(0, 60, 40)$.

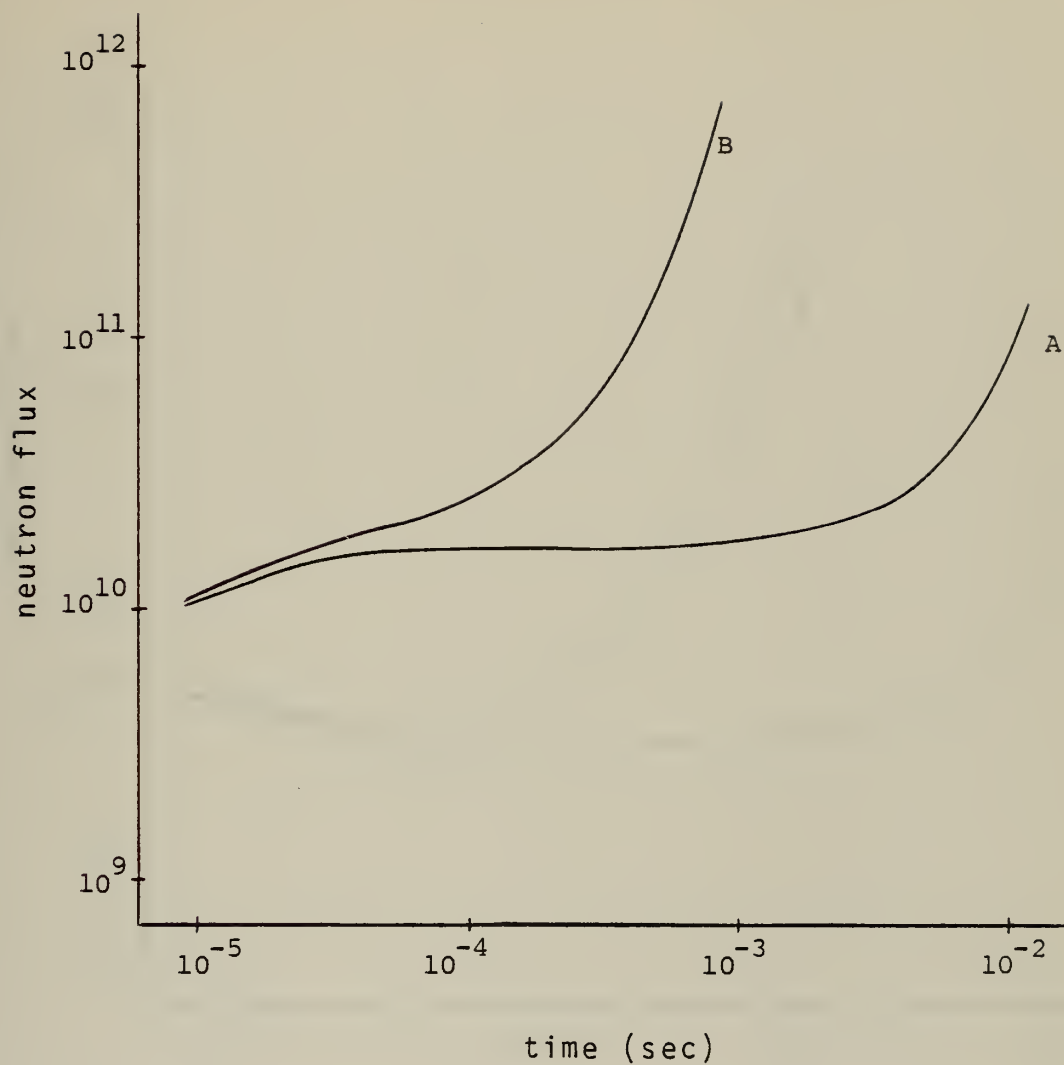


Figure 18. Linear and nonlinear fluxes at (0,0,0) due to a 50 dollar/sec local perturbation at (0,60,40).

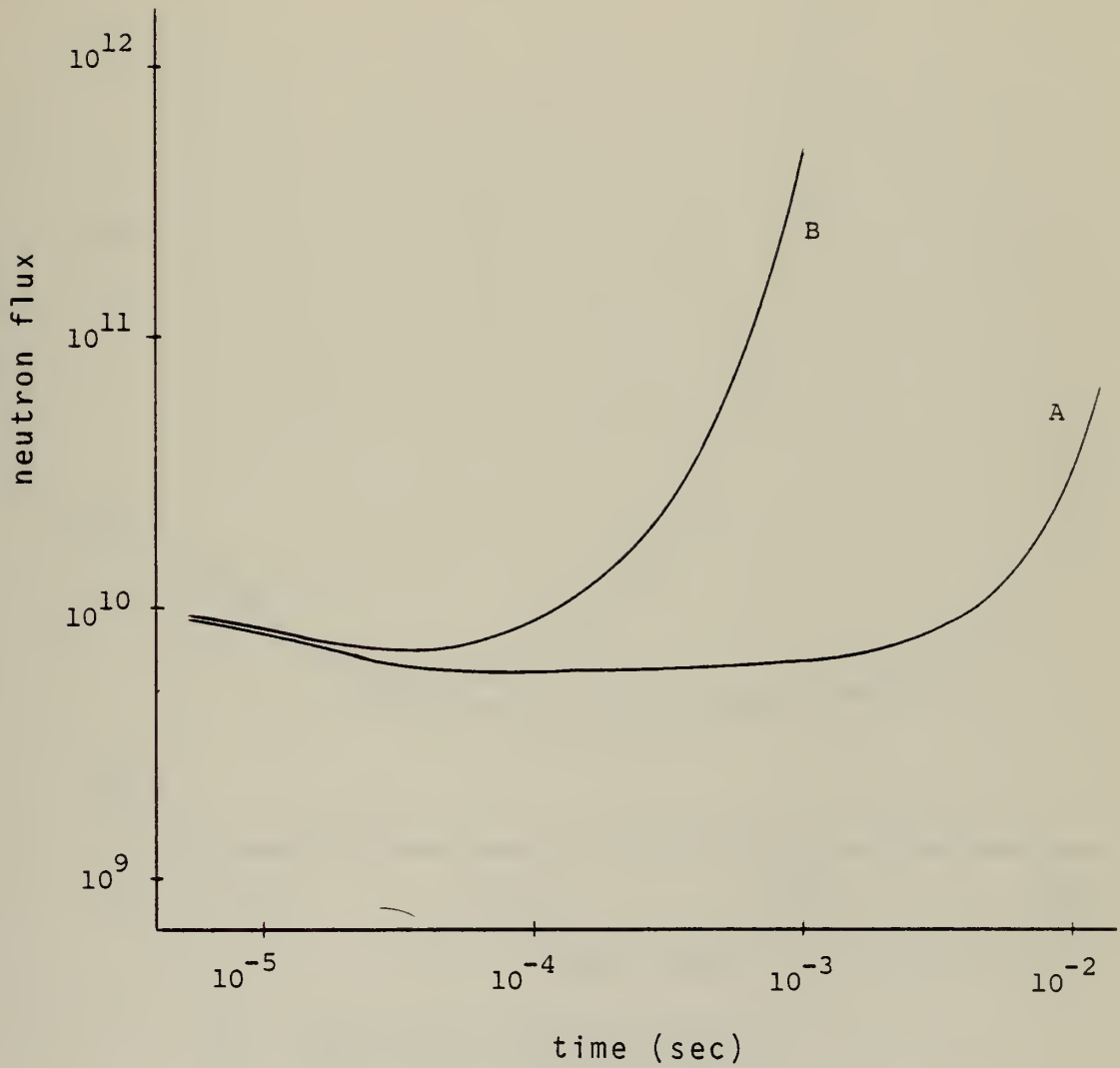


Figure 19. Linear and nonlinear fluxes at (60,0,0) due to a 50 dollar/sec local perturbation at (0,60,40).

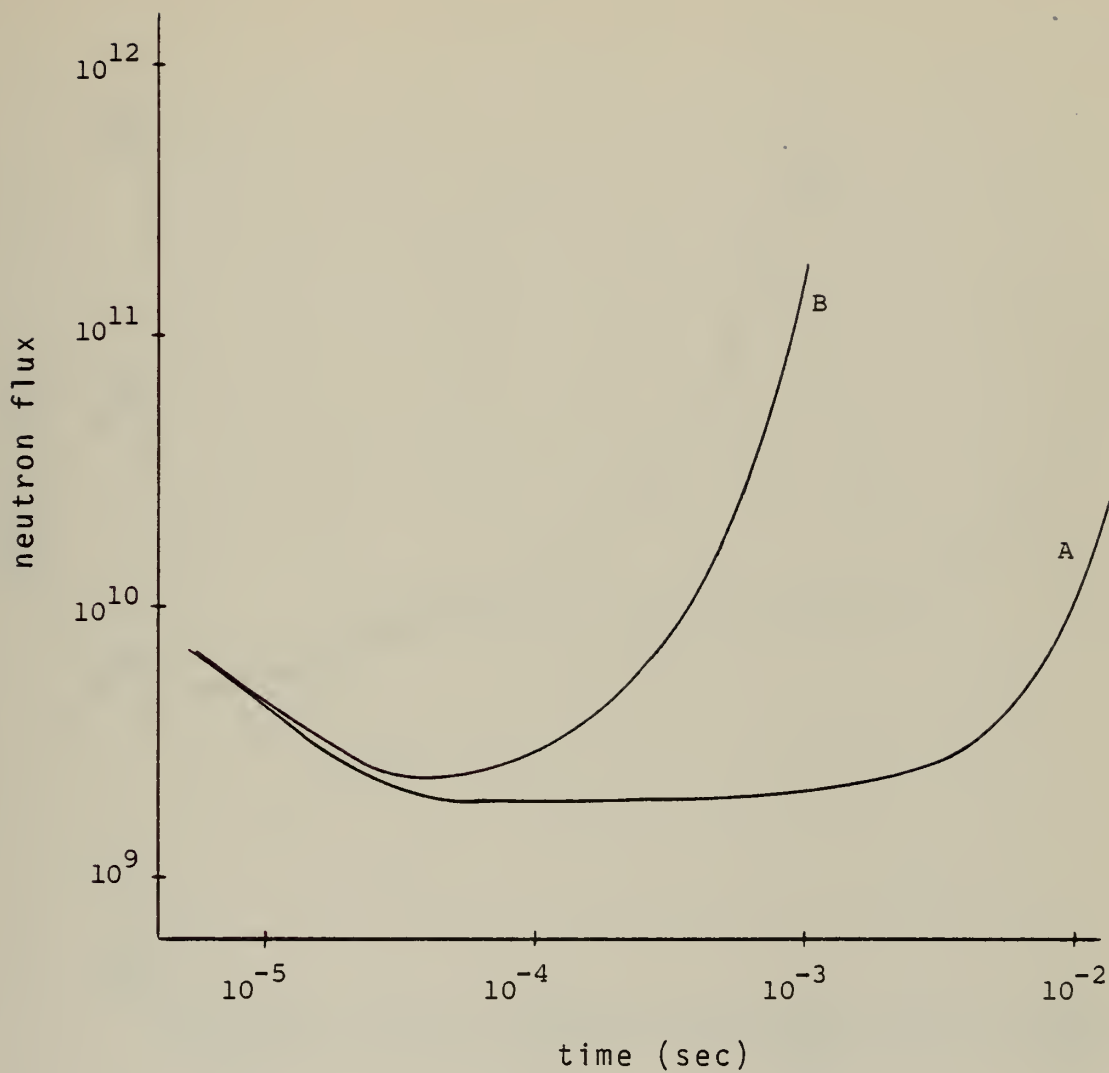
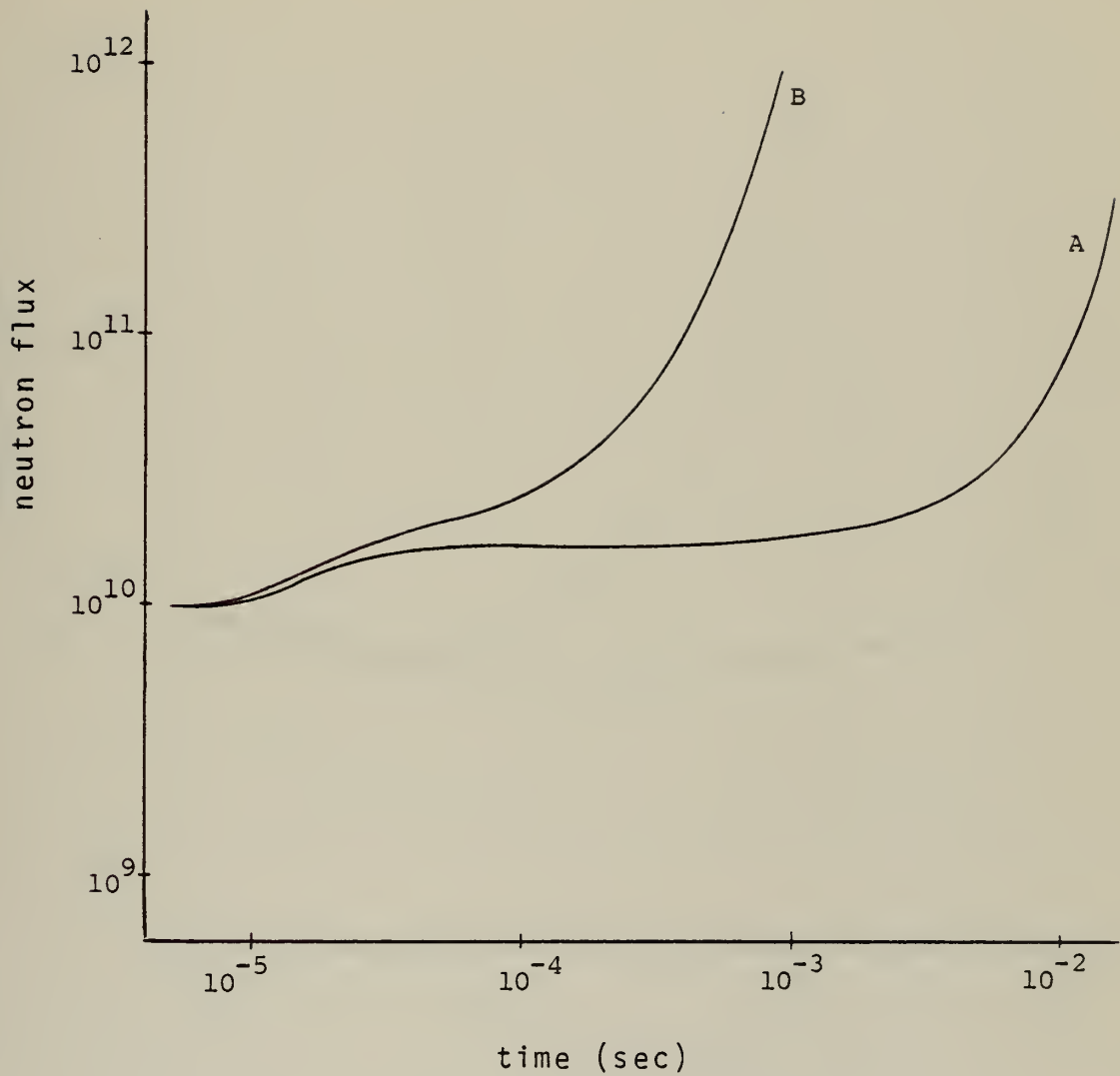
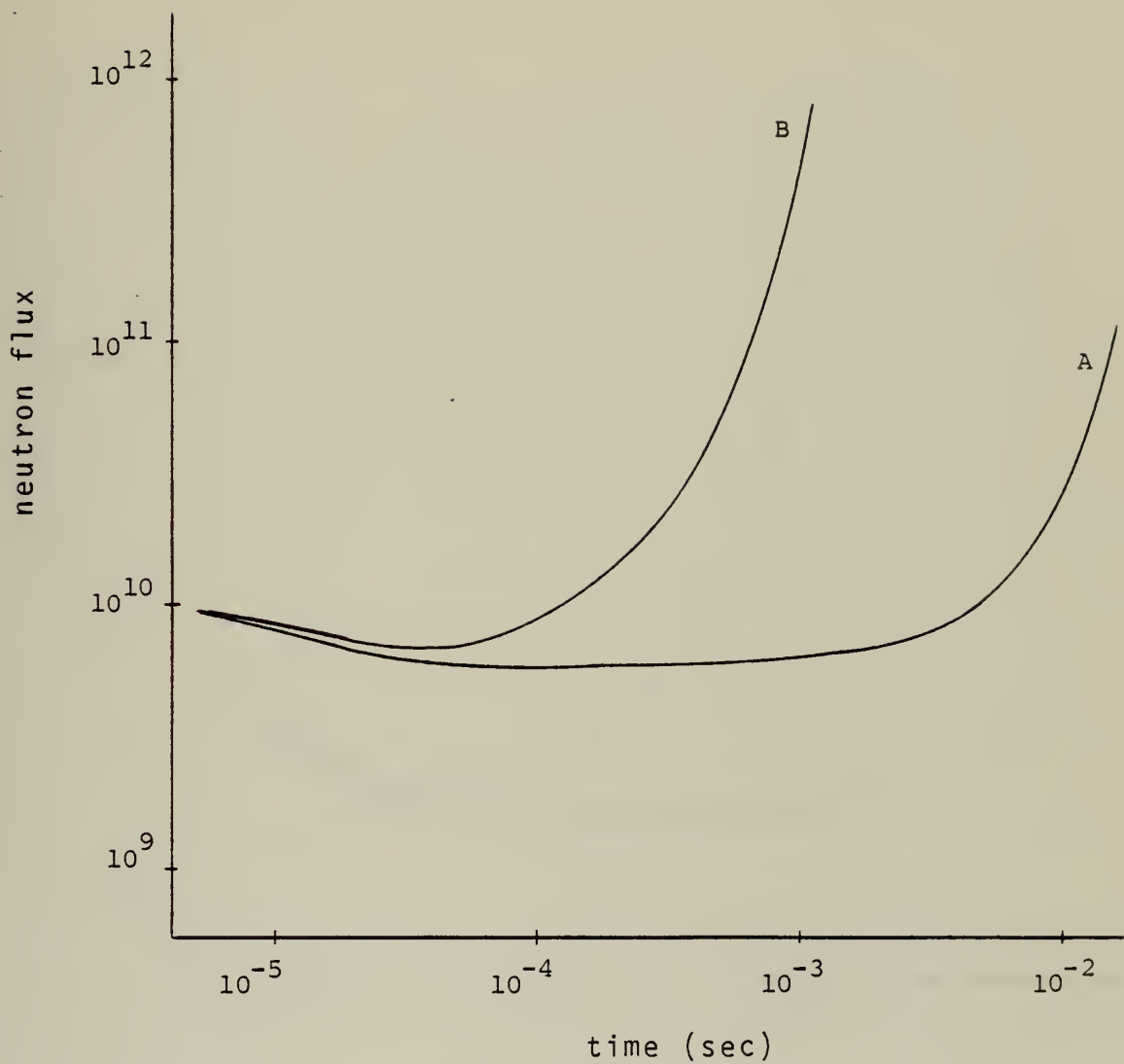


Figure 20. Linear and nonlinear fluxes at $(-60, 0, 80)$ due to a 50 dollar/sec local perturbation at $(0, 60, 40)$.



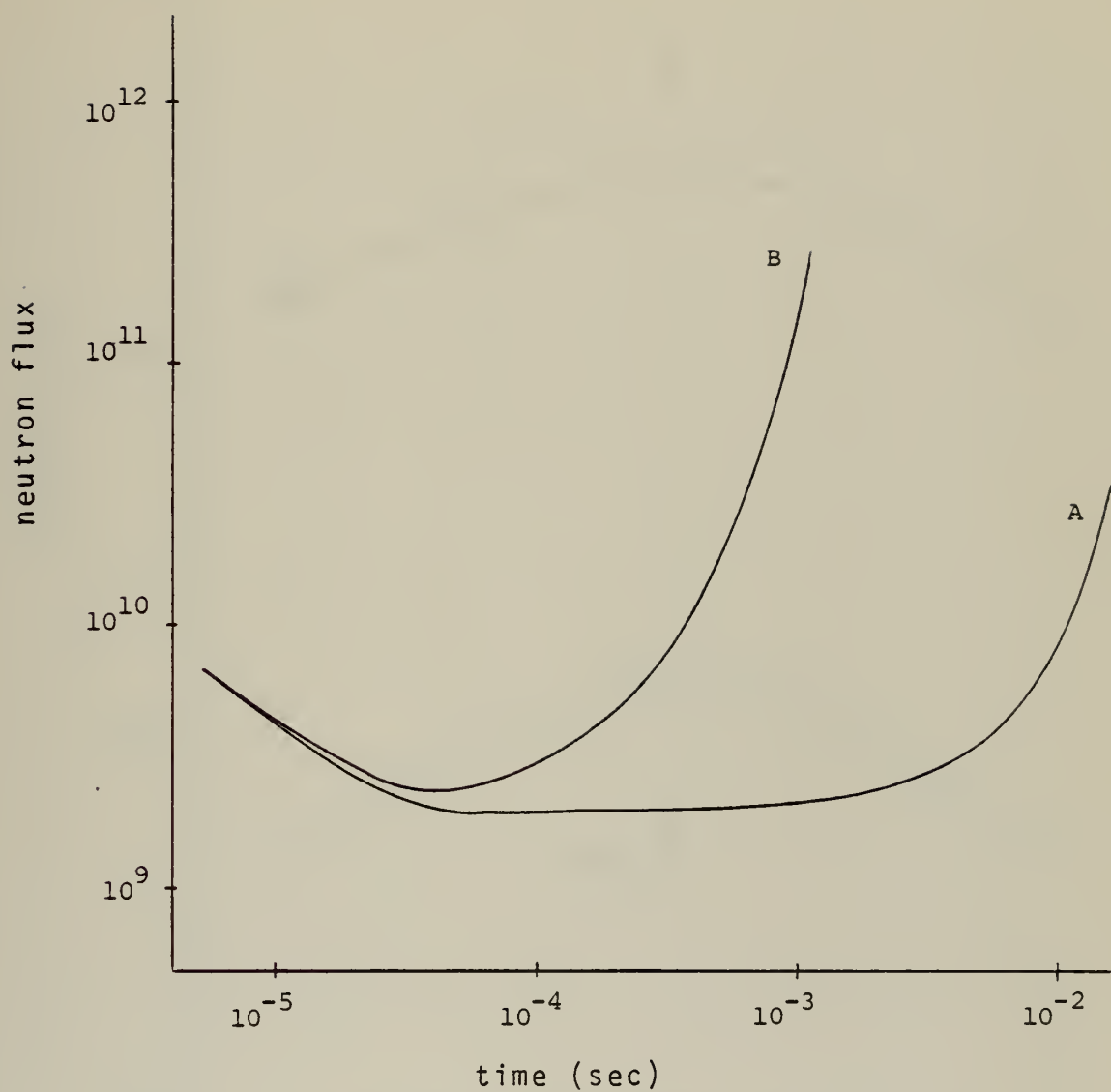
A - nonlinear neutron response
 B - linear neutron response

Figure 21. Linear and nonlinear fluxes at (0,0,0) due to a 10 dollar/sec local perturbation at (0,60,40).



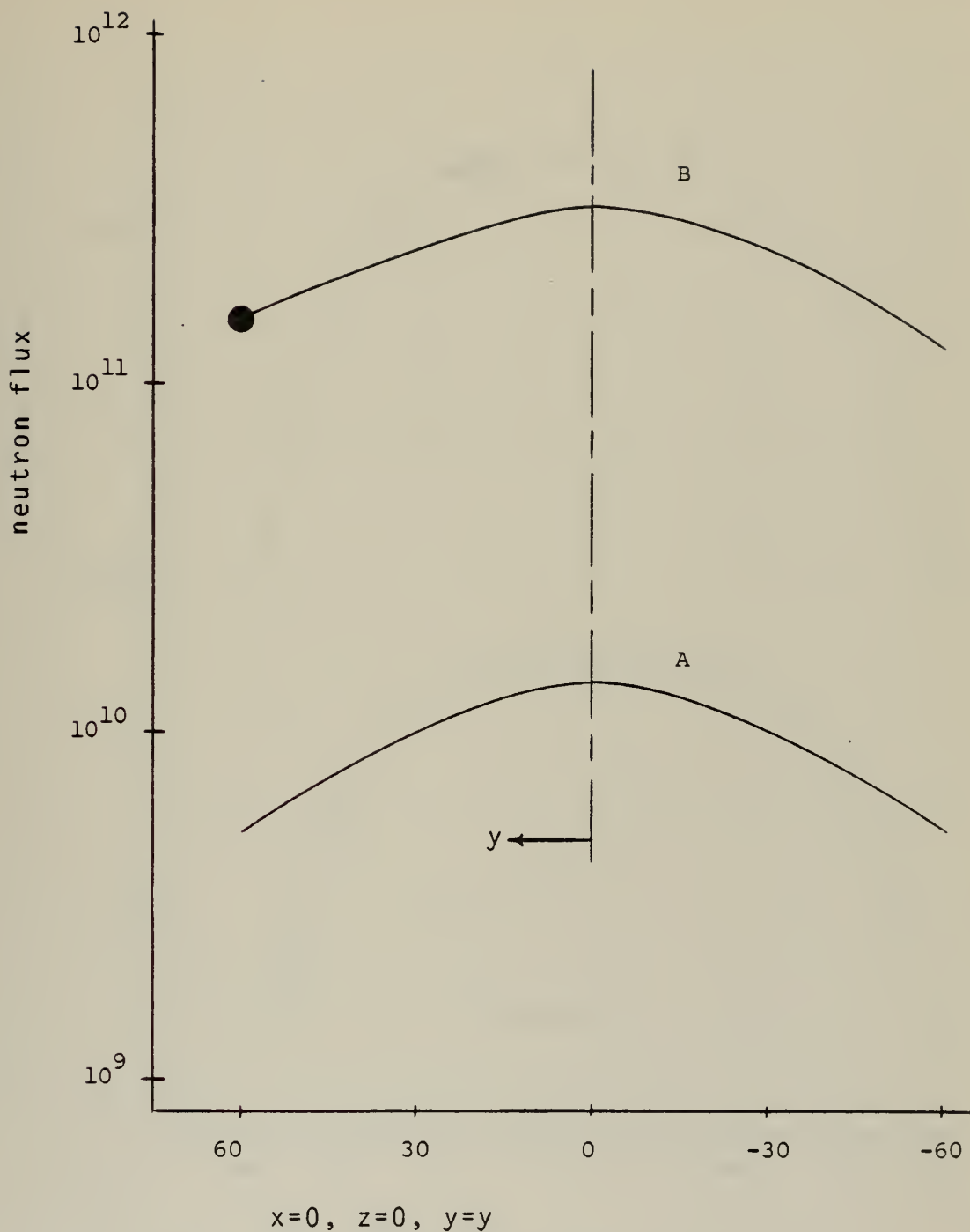
A - nonlinear neutron response
B - linear neutron response

Figure 22. Linear and nonlinear fluxes at (60,0,0) due to a 10 dollar/sec local perturbation at (0,60,40).



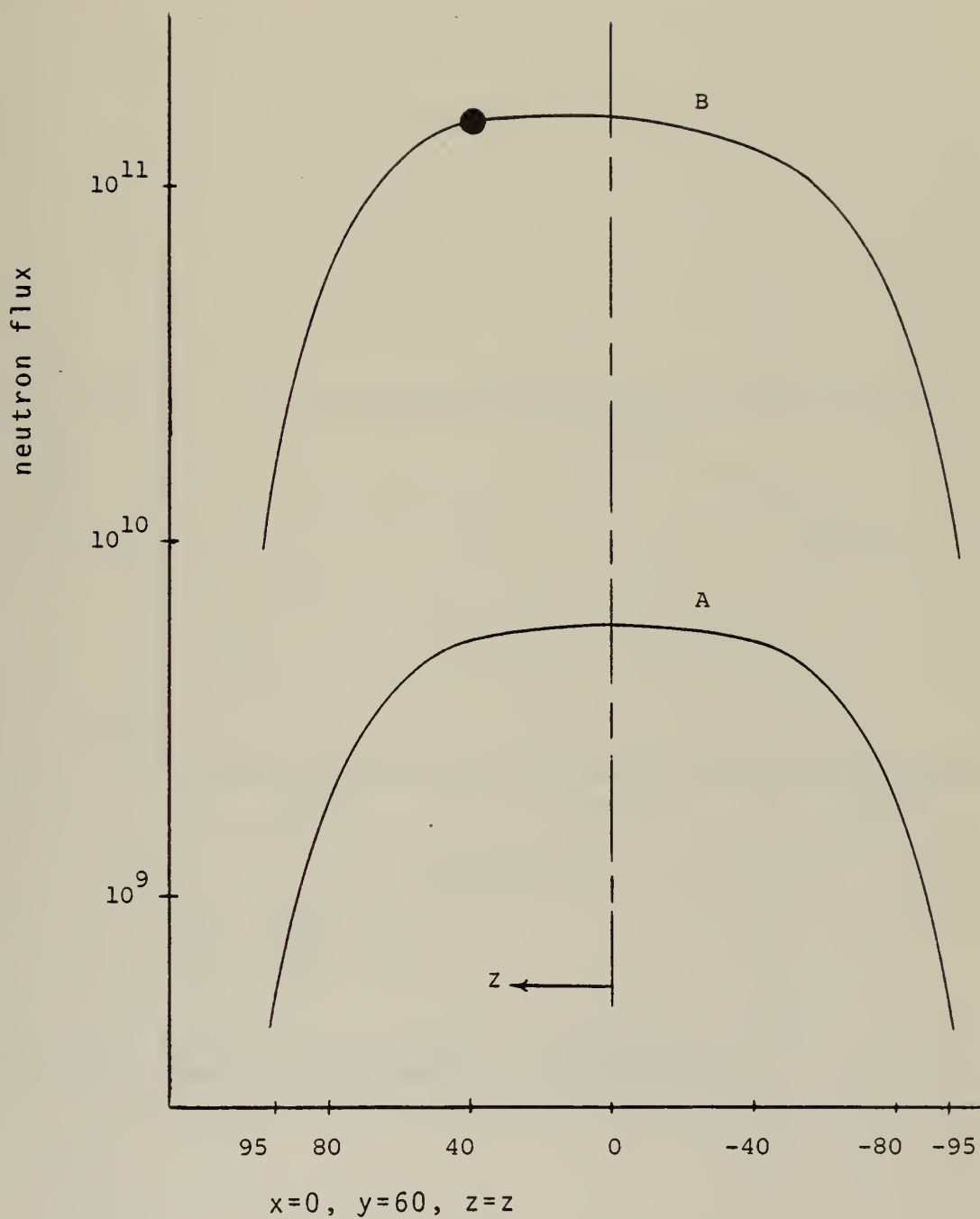
A - nonlinear neutron response
 B - linear neutron response

Figure 23. Linear and nonlinear fluxes at $(-60, 0, 80)$ due to a 10 dollar/sec local perturbation at $(0, 60, 40)$.



- - point of local perturbation
- A - neutron flux at steady-state
- B - neutron flux under local perturbation

Figure 24. Radial flux distribution for the steady state and 100 dollar/sec local perturbation.



- - point of local perturbation
- A - axial flux distribution during steady state
- B - axial flux distribution during perturbation

Figure 25. Axial flux distribution for the steady state and 100 dollar/sec local perturbation.

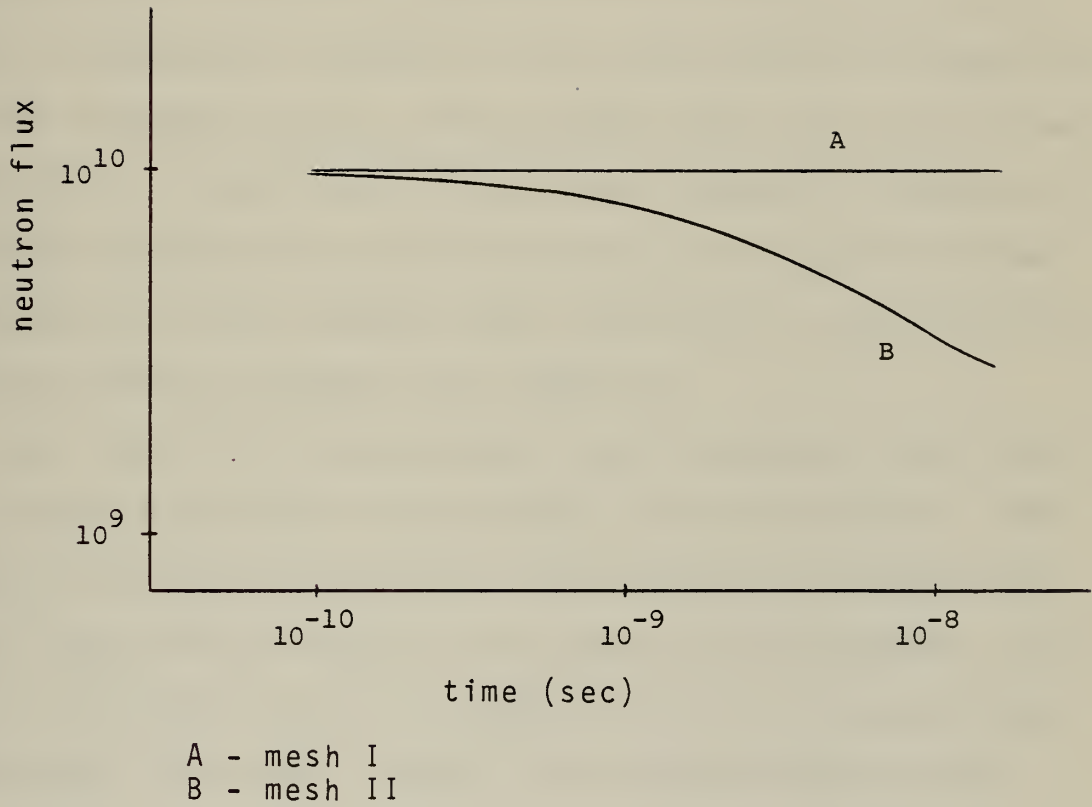


Figure 26. Early time history of the neutron flux at core center using mesh I and mesh II.

and delayed neutrons on the flux. To obtain an idea of the difference in magnitude between the linear and nonlinear flux for the 100 dollar local perturbation, at time $t = 10^{-3}$ second, the linear case predicted a neutron flux at the core center of 2.024×10^{12} neutrons per cm^2 per sec. The nonlinear reactor predicted 1.85×10^{10} neutrons per cm^2 per sec. Although the numbers are not sufficiently accurate due to the crudeness of the finite element mesh, the significant difference in the order of magnitude between the linear and nonlinear neutron flux leads to the belief that the three-dimensional finite element model utilized is predicting the correct trend of neutron flux behavior.

The radial flux distribution for the steady state shown in figure 24 portrays the expected flux distribution. The radial flux distributions for the local perturbation case show a skew distribution at the point of perturbation. The axial flux distribution at steady state is very much symmetric about the center. Again, the expected skew at the point of perturbation is there, as shown in figure 25. Figure 26 gives an indication that the Σ_f^* for finer meshes is higher. However, curve B of figure 26 should be extended to longer times to verify this hypothesis. Curve B, as plotted in figure 26, used four hours of computer time employing the H-compiler of the IBM 360/67.

VI. CONCLUSIONS AND RECOMMENDATIONS

The finite element mesh employed here is crude, and, thus, results that were obtained should be considered as positive indicators rather than numerically conclusive facts. The trend of neutron flux behavior is the major thrust of this work. From the results obtained, it is concluded that the expected patterns of neutron behavior as predicted by the three-dimensional finite element model used do occur. These patterns were best demonstrated by the differences between the linear and nonlinear flux responses and by the spatial flux distribution at steady state and during local perturbation. The three-dimensional quadratic finite element model utilized should produce better results by resorting to a finer mesh. The draw-back to a finer mesh, of course, is the significant increase in computer time and storage requirements. Once accurate results are obtained through finer element meshes, comparisons between three- and two-dimensional models can be attempted.

A mesh generator was not developed for this problem. It is recommended that this be done to ease the transition from one mesh to another and to minimize human error. In addition, a similar calculation using a three-dimensional linear element should be performed to corroborate the results obtained here. It should be particularly noted that this type of problem is highly sensitive to the fission cross-section.

In the search for Σ_f^* , it was necessary to adjust Σ_f to the sixth decimal place. There is no exact method of deriving Σ_f^* due to the highly nonlinear aspect of the problem; therefore, trial and error must be used. Finally, the Gaussian quadrature used to determine the element matrices should be investigated. The cause of the differences in values obtained by using different number of integration points should be established. Was it due to the integration points, the coordinate transformation, or the element shape functions? This is an important question upon which future numerical results could be based.

APPENDIX A

MESH I CONNECTIVITY AND COORDINATES

The connectivity matrix for the 128-element mesh is as follows:

ELEMENT NUMBER	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	3	12	11	10	2	26	28	27	35	37	46	45	44	36
2	1	4	14	13	12	3	26	29	28	35	38	48	47	46	37
3	1	5	15	14	13	4	26	30	29	35	39	50	49	48	38
4	1	6	16	15	14	5	26	31	30	35	40	52	51	50	39
5	1	7	17	16	15	6	26	32	31	35	41	54	53	52	40
6	1	8	18	17	16	7	26	33	32	35	42	56	55	54	41
7	1	9	19	18	17	8	26	34	33	35	43	58	57	56	42
8	1	10	20	19	18	9	26	35	34	35	36	44	43	42	43
9	1	11	21	20	19	10	26	36	35	35	37	80	79	78	70
10	3	37	45	44	43	36	60	62	61	69	72	82	81	80	72
11	3	38	46	45	44	37	60	63	62	69	73	84	83	82	73
12	3	39	47	46	45	38	60	64	63	69	74	86	85	84	74
13	3	40	48	47	46	39	60	65	64	69	75	88	87	86	75
14	3	41	49	48	47	40	60	66	65	69	76	90	89	88	76
15	3	42	50	49	48	41	60	67	66	69	77	92	91	90	77
16	3	43	51	50	49	42	60	68	67	69	78	93	92	91	78
17	3	44	52	51	50	43	60	69	68	69	79	94	93	92	79
18	3	45	53	52	51	44	60	70	69	69	80	95	94	93	80
19	3	46	54	53	52	45	60	71	70	425	427	436	435	434	426
20	3	47	55	54	53	46	60	72	71	425	428	438	437	436	427
21	3	48	56	55	54	47	60	73	72	425	429	440	439	438	428
22	3	49	57	56	55	48	60	74	73	425	430	442	441	440	429
23	3	50	58	57	56	49	60	75	74	425	431	444	443	442	430
24	3	51	59	58	57	50	60	76	75	425	432	446	445	444	431
25	3	52	60	59	58	51	60	77	76	425	433	448	447	446	432
26	3	53	61	60	59	52	60	78	77	425	434	451	449	448	433
27	3	54	62	61	60	53	60	79	78	425	435	453	452	451	434
28	3	55	63	62	61	54	60	80	79	425	436	454	453	452	435
29	3	56	64	63	62	55	60	81	80	425	437	456	455	454	436
30	3	57	65	64	63	56	60	82	81	425	438	457	456	455	437
31	3	58	66	65	64	57	60	83	82	425	439	458	457	456	438
32	3	59	67	66	65	58	60	84	83	425	440	459	458	457	439
33	3	60	68	67	66	59	60	85	84	425	441	460	459	458	440
34	3	61	69	68	67	60	60	86	85	425	442	461	460	459	441
35	3	62	70	69	68	61	60	87	86	425	443	462	461	460	442
36	3	63	71	70	69	62	60	88	87	425	444	463	462	461	443
37	3	64	72	71	70	63	60	89	88	425	445	464	463	462	444
38	3	65	73	72	71	64	60	90	89	425	446	465	464	463	445
39	3	66	74	73	72	65	60	91	90	425	447	466	465	464	446
40	3	67	75	74	73	66	60	92	91	425	448	467	466	465	447
41	3	68	76	75	74	67	60	93	92	425	449	468	467	466	448
42	3	69	77	76	75	68	60	94	93	425	450	469	468	467	449
43	3	70	78	77	76	69	60	95	94	425	451	470	469	468	450
44	3	71	79	78	77	70	60	96	95	425	452	471	470	469	451
45	3	72	80	79	78	71	60	97	96	425	453	472	471	470	452
46	3	73	81	80	79	72	60	98	97	425	454	473	472	471	453
47	3	74	82	81	80	73	60	99	98	425	455	474	473	472	454
48	3	75	83	82	81	74	60	100	99	425	456	475	474	473	455
49	3	76	84	83	82	75	60	101	100	425	457	476	475	474	456
50	3	77	85	84	83	76	60	102	101	425	458	477	476	475	457
51	3	78	86	85	84	77	60	103	102	425	459	478	477	476	458
52	3	79	87	86	85	78	60	104	103	425	460	479	478	477	459
53	3	80	88	87	86	79	60	105	104	425	461	480	479	478	460
54	3	81	89	88	87	80	60	106	105	425	462	481	480	479	461
55	3	82	90	89	88	81	60	107	106	425	463	482	481	480	462
56	3	83	91	90	89	82	60	108	107	425	464	483	482	481	463
57	3	84	92	91	90	83	60	109	108	425	465	484	483	482	464
58	3	85	93	92	91	84	60	110	109	425	466	485	484	483	465
59	3	86	94	93	92	85	60	111	110	425	467	486	485	484	466
60	3	87	95	94	93	86	60	112	111	425	468	487	486	485	467
61	3	88	96	95	94	87	60	113	112	425	469	488	487	486	468
62	3	89	97	96	95	88	60	114	113	425	470	489	488	487	469
63	3	90	98	97	96	89	60	115	114	425	471	490	489	488	470
64	3	91	99	98	97	90	60	116	115	425	472	491	490	489	471
65	3	92	100	99	98	91	60	117	116	425	473	492	491	490	472
66	3	93	101	100	99	92	60	118	117	425	474	493	492	491	473
67	3	94	102	101	100	93	60	119	118	425	475	494	493	492	474
68	3	95	103	102	101	94	60	120	119	425	476	495	494	493	475
69	3	96	104	103	102	95	60	121	120	425	477	496	495	494	476
70	3	97	105	104	103	96	60	122	121	425	478	497	496	495	477
71	3	98	106	105	104	97	60	123	122	425	479	498	497	496	478
72	3	99	107	106	105	98	60	124	123	425	480	499	498	497	479
73	3	100	108	107	106	99	60	125	124	425	481	500	499	498	480
74	3	101	109	108	107	100	60	126	125	425	482	501	500	499	481
75	3	102	110	109	108	101	60	127	126	425	483	502	501	500	482
76	3	103	111	110	109	102	60	128	127	425	484	503	502	501	483
77	3	104	112	111	110	103	60	129	128	425	485	504	503	502	484
78	3	105	113	112	111	104	60	130	129	425	486	505	504	503	485
79	3	106	114	113	112	105	60	131	130	425	487	506	505	504	486
80	3	107	115	114	113	106	60	132	131	425	488	507	506	505	487
81	3	108	116	115	114	107	60	133	132	425	489	508	507	506	488
82	3	109	117	116	115	108	60	134	133	425	490	509	508	507	489
83	3	110	118	117	116	109	60	135	134	425	491	510	509	508	490
84	3	111	119	118	117	110	60	136	135	425	492	511	510	509	491
85	3	112	120	119	118	111	60	137	136	425	493	512	511	510	492
86	3	113	121	120	119	112	60	138	137	425	494	513	512	511	493
87	3	114	122	121	120	113	60	139	138	425	495	514	513	512	494
88	3	115	123	122	121	114	60	140	139	425	496	515	514	513	495
89	3	116	124	123	122	115	60	141	140	425	497	516	515	514	496
90	3	117	125	124	123	116	60	142	141	425	498	517	516	515	497
91	3	118	126	125	124	117	60	143	142	425	499	518	517	516	498
92	3	119	127	126	125	118	60	144	143	425	500	519	518	517	499
93	3	120	128	127	126	119	60	145	144	425	501	520	519	518	500
94	3	121	129	128	127	120	60	146	145	425	502	521	520	519	501
95	3	122	130	129	128	121	60	147	146	425	503	522	521	520	502
96	3	123	131	130	129	122	60	148	147	425	504	523	522	521	503
97	3	124	132	131	130	123	60	149	148	425	505	524	523	522	504
98	3	125	133	132	131	124									

[illegible]

The coordinates of the 128-element mesh are as follows:

NODE NUMBER	X	Y	Z
1	0.0	0.0	80.00
2	0.0	30.00	80.00
3	21.21	21.21	80.00
4	30.00	0.0	80.00
5	21.21	-21.21	80.00
6	0.0	-30.00	80.00
7	-21.21	-21.21	80.00
8	-30.00	0.0	80.00
9	-21.21	21.21	80.00
10	0.0	60.00	80.00
11	22.96	55.43	80.00
12	42.43	42.43	80.00
13	55.43	22.96	80.00
14	60.00	0.0	80.00
15	55.43	-22.96	80.00
16	42.43	-42.43	80.00
17	22.96	-55.43	80.00
18	0.0	-60.00	80.00
19	-22.96	-55.43	80.00
20	-42.43	-42.43	80.00
21	-55.43	-22.96	80.00
22	-60.00	0.0	80.00
23	-55.43	22.96	80.00
24	-42.43	42.43	80.00
25	-22.96	55.43	80.00
26	0.0	60.00	40.00
27	0.0	42.43	40.00
28	42.43	0.0	40.00
29	60.00	0.0	40.00
30	42.43	-42.43	40.00
31	0.0	-60.00	40.00
32	-42.43	-42.43	40.00
33	-60.00	0.0	40.00
34	-55.43	22.96	40.00
35	-42.43	42.43	40.00
36	-22.96	55.43	40.00
37	0.0	60.00	0.0
38	21.21	21.21	0.0
39	30.00	0.0	0.0
40	21.21	-21.21	0.0
41	0.0	-30.00	0.0
42	-21.21	-21.21	0.0
43	-30.00	0.0	0.0
44	-21.21	21.21	0.0

45	00.00	00.00	00.00
46	00.00	00.00	00.00
47	00.00	00.00	00.00
48	00.00	00.00	00.00
49	00.00	00.00	00.00
50	00.00	00.00	00.00
51	00.00	00.00	00.00
52	00.00	00.00	00.00
53	00.00	00.00	00.00
54	00.00	00.00	00.00
55	00.00	00.00	00.00
56	00.00	00.00	00.00
57	00.00	00.00	00.00
58	00.00	00.00	00.00
59	00.00	00.00	00.00
60	00.00	00.00	00.00
61	00.00	00.00	00.00
62	00.00	00.00	00.00
63	00.00	00.00	00.00
64	00.00	00.00	00.00
65	00.00	00.00	00.00
66	00.00	00.00	00.00
67	00.00	00.00	00.00
68	00.00	00.00	00.00
69	00.00	00.00	00.00
70	00.00	00.00	00.00
71	00.00	00.00	00.00
72	00.00	00.00	00.00
73	00.00	00.00	00.00
74	00.00	00.00	00.00
75	00.00	00.00	00.00
76	00.00	00.00	00.00
77	00.00	00.00	00.00
78	00.00	00.00	00.00
79	00.00	00.00	00.00
80	00.00	00.00	00.00
81	00.00	00.00	00.00
82	00.00	00.00	00.00
83	00.00	00.00	00.00
84	00.00	00.00	00.00
85	00.00	00.00	00.00
86	00.00	00.00	00.00
87	00.00	00.00	00.00
88	00.00	00.00	00.00
89	00.00	00.00	00.00
90	00.00	00.00	00.00
91	00.00	00.00	00.00
92	00.00	00.00	00.00

55.43	55.43	55.43	55.43
22.96	22.96	22.96	22.96
-42.43	-42.43	-42.43	-42.43
-55.43	-55.43	-55.43	-55.43
-55.43	-55.43	-55.43	-55.43
-42.96	-42.96	-42.96	-42.96
-55.43	-55.43	-55.43	-55.43
-42.96	-42.96	-42.96	-42.96
22.96	22.96	22.96	22.96
42.43	42.43	42.43	42.43
55.43	55.43	55.43	55.43
00.00	00.00	00.00	00.00
60.43	60.43	60.43	60.43
42.43	42.43	42.43	42.43
-42.43	-42.43	-42.43	-42.43
-60.43	-60.43	-60.43	-60.43
-42.43	-42.43	-42.43	-42.43
42.43	42.43	42.43	42.43
30.00	30.00	30.00	30.00
21.21	21.21	21.21	21.21
21.21	21.21	21.21	21.21
-30.00	-30.00	-30.00	-30.00
-21.21	-21.21	-21.21	-21.21
-21.21	-21.21	-21.21	-21.21
21.21	21.21	21.21	21.21
60.43	60.43	60.43	60.43
55.43	55.43	55.43	55.43
42.96	42.96	42.96	42.96
22.96	22.96	22.96	22.96
-42.43	-42.43	-42.43	-42.43
-55.43	-55.43	-55.43	-55.43
-55.43	-55.43	-55.43	-55.43
-42.96	-42.96	-42.96	-42.96
22.96	22.96	22.96	22.96
42.43	42.43	42.43	42.43

22.96	22.96	22.96	22.96
55.43	55.43	55.43	55.43
55.43	55.43	55.43	55.43
55.43	55.43	55.43	55.43
42.96	42.96	42.96	42.96
22.96	22.96	22.96	22.96
-42.43	-42.43	-42.43	-42.43
-55.43	-55.43	-55.43	-55.43
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22.96	22.96	22.96	22.96
42.43	42.43	42.43	42.43
60.43	60.43	60.43	60.43
42.43	42.43	42.43	42.43
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-60.43	-60.43	-60.43	-60.43
-42.43	-42.43	-42.43	-42.43
00.00	00.00	00.00	00.00
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30.00	30.00	30.00	30.00
21.21	21.21	21.21	21.21
-21.21	-21.21	-21.21	-21.21
-30.00	-30.00	-30.00	-30.00
-21.21	-21.21	-21.21	-21.21
22.96	22.96	22.96	22.96
42.43	42.43	42.43	42.43
55.43	55.43	55.43	55.43
60.43	60.43	60.43	60.43
42.96	42.96	42.96	42.96
22.96	22.96	22.96	22.96
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-55.43	-55.43	-55.43	-55.43
-55.43	-55.43	-55.43	-55.43
-42.96	-42.96	-42.96	-42.96
22.96	22.96	22.96	22.96
42.43	42.43	42.43	42.43

93	-22.96	55.43	-80.00	00
94	16.80	75.00	-80.00	00
95	38.10	71.60	-80.00	00
96	53.03	62.90	-80.00	00
97	62.80	53.03	-80.00	00
98	71.30	38.60	-80.00	00
99	75.00	17.00	-80.00	00
100	71.30	17.00	-80.00	00
101	62.80	38.60	-80.00	00
102	53.03	62.90	-80.00	00
103	38.10	71.80	-80.00	00
104	17.10	75.00	-80.00	00
105	17.10	71.80	-80.00	00
106	16.90	75.00	-80.00	00
107	38.50	71.80	-80.00	00
108	53.03	62.90	-80.00	00
109	62.80	53.03	-80.00	00
110	71.30	38.60	-80.00	00
111	75.00	17.10	-80.00	00
112	71.30	17.00	-80.00	00
113	62.80	38.60	-80.00	00
114	53.03	71.80	-80.00	00
115	38.10	75.00	-80.00	00
116	17.10	71.80	-80.00	00
117	17.10	75.00	-80.00	00
118	16.90	71.80	-80.00	00
119	38.50	75.00	-80.00	00
120	53.03	62.90	-80.00	00
121	62.80	53.03	-80.00	00
122	71.30	38.60	-80.00	00
123	75.00	17.10	-80.00	00
124	71.30	17.00	-80.00	00
125	62.80	38.60	-80.00	00
126	53.03	71.80	-80.00	00
127	38.10	75.00	-80.00	00
128	17.10	71.80	-80.00	00
129	17.10	75.00	-80.00	00
130	16.90	71.80	-80.00	00
131	38.50	75.00	-80.00	00
132	53.03	62.90	-80.00	00
133	62.80	53.03	-80.00	00
134	71.30	38.60	-80.00	00
135	75.00	17.10	-80.00	00
136	71.30	17.00	-80.00	00
137	62.80	38.60	-80.00	00
138	53.03	71.80	-80.00	00
139	38.10	75.00	-80.00	00
140	17.10	71.80	-80.00	00
	16.90	75.00	-80.00	00

141	-38.50	-62.80	0.0
142	-53.03	-53.60	0.0
143	-62.70	-17.10	0.0
144	-71.30	0.0	0.0
145	-75.00	17.10	0.0
146	-71.30	38.03	0.0
147	-62.90	53.80	0.0
148	-53.03	62.30	0.0
149	-38.60	71.30	0.0
150	-17.30	75.00	80.00
151	16.80	71.60	80.00
152	38.10	62.90	80.00
153	53.03	53.03	80.00
154	62.80	38.60	80.00
155	71.30	17.00	80.00
156	75.00	0.0	80.00
157	71.30	17.00	80.00
158	62.80	-38.60	80.00
159	53.03	-53.03	80.00
160	38.10	-62.90	80.00
161	17.00	-71.80	80.00
162	0.0	-75.00	80.00
163	-16.90	-71.80	80.00
164	-38.50	-62.03	80.00
165	-53.03	-38.60	80.00
166	-62.70	-17.10	80.00
167	-71.30	0.0	80.00
168	-75.00	17.10	80.00
169	-71.30	38.03	80.00
170	-62.90	53.80	80.00
171	-53.03	62.30	80.00
172	-38.60	71.30	80.00
173	-17.30	75.00	95.00
174	0.0	60.00	95.00
175	0.0	42.043	95.00
176	42.043	0.0	95.00
177	60.00	42.043	95.00
178	42.043	-42.043	95.00
179	0.0	-60.00	95.00
180	-42.043	-42.043	95.00
181	-60.00	42.043	95.00
182	-42.043	0.0	110.00
183	0.0	0.0	110.00
184	0.0	30.21	110.00
185	21.21	21.21	110.00
186	30.00	0.0	110.00
187	21.21	-21.21	110.00
188	21.21		

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0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0	6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7.0	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8.0	8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8	8.9	9.0	9.1	9.2	9.3	9.4	9.5	9.6	9.7	9.8	9.9	10.0	10.1	10.2	10.3	10.4	10.5	10.6	10.7	10.8	10.9	11.0	11.1	11.2	11.3	11.4	11.5	11.6	11.7	11.8	11.9	12.0	12.1	12.2	12.3	12.4	12.5	12.6	12.7	12.8	12.9	13.0	13.1	13.2	13.3	13.4	13.5	13.6	13.7	13.8	13.9	14.0	14.1	14.2	14.3	14.4	14.5	14.6	14.7	14.8	14.9	15.0	15.1	15.2	15.3	15.4	15.5	15.6	15.7	15.8	15.9	16.0	16.1	16.2	16.3	16.4	16.5	16.6	16.7	16.8	16.9	17.0	17.1	17.2	17.3	17.4	17.5	17.6	17.7	17.8	17.9	18.0	18.1	18.2	18.3	18.4	18.5	18.6	18.7	18.8	18.9	19.0	19.1	19.2	19.3	19.4	19.5	19.6	19.7	19.8	19.9	20.0	20.1	20.2	20.3	20.4	20.5	20.6	20.7	20.8	20.9	21.0	21.1	21.2	21.3	21.4	21.5	21.6	21.7	21.8	21.9	22.0	22.1	22.2	22.3	22.4	22.5	22.6	22.7	22.8	22.9	23.0	23.1	23.2	23.3	23.4	23.5	23.6	23.7	23.8	23.9	24.0	24.1	24.2	24.3	24.4	24.5	24.6	24.7	24.8	24.9	25.0	25.1	25.2	25.3	25.4	25.5	25.6	25.7	25.8	25.9	26.0	26.1	26.2	26.3	26.4	26.5	26.6	26.7	26.8	26.9	27.0	27.1	27.2	27.3	27.4	27.5	27.6	27.7	27.8	27.9	28.0	28.1	28.2	28.3	28.4	28.5	28.6	28.7	28.8	28.9	29.0	29.1	29.2	29.3	29.4	29.5	29.6	29.7	29.8	29.9	30.0	30.1	30.2	30.3	30.4	30.5	30.6	30.7	30.8	30.9	31.0	31.1	31.2	31.3	31.4	31.5	31.6	31.7	31.8	31.9	32.0	32.1	32.2	32.3	32.4	32.5	32.6	32.7	32.8	32.9	33.0	33.1	33.2	33.3	33.4	33.5	33.6	33.7	33.8	33.9	34.0	34.1	34.2	34.3	34.4	34.5	34.6	34.7	34.8	34.9	35.0	35.1	35.2	35.3	35.4	35.5	35.6	35.7	35.8	35.9	36.0	36.1	36.2	36.3	36.4	36.5	36.6	36.7	36.8	36.9	37.0	37.1	37.2	37.3	37.4	37.5	37.6	37.7	37.8	37.9	38.0	38.1	38.2	38.3	38.4	38.5	38.6	38.7	38.8	38.9	39.0	39.1	39.2	39.3	39.4	39.5	39.6	39.7	39.8	39.9	40.0	40.1	40.2	40.3	40.4	40.5	40.6	40.7	40.8	40.9	41.0	41.1	41.2	41.3	41.4	41.5	41.6	41.7	41.8	41.9	42.0	42.1	42.2	42.3	42.4	42.5	42.6	42.7	42.8	42.9	43.0	43.1	43.2	43.3	43.4	43.5	43.6	43.7	43.8	43.9	44.0	44.1	44.2	44.3	44.4	44.5	44.6	44.7	44.8	44.9	45.0	45.1	45.2	45.3	45.4	45.5	45.6	45.7	45.8	45.9	46.0	46.1	46.2	46.3	46.4	
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237	63.683	63.64	110.00
238	74.157	50.00	110.00
239	88.217	34.44	110.00
240	90.000	17.56	110.00
241	88.217	-17.56	110.00
242	83.153	-34.44	110.00
243	74.834	-50.00	110.00
244	63.640	-63.64	110.00
245	50.000	-74.83	110.00
246	34.446	-83.15	110.00
247	17.556	-88.20	110.00
248	10.000	-90.00	110.00
249	-17.566	-88.27	110.00
250	-34.444	-83.15	110.00
251	-50.000	-74.83	110.00
252	-63.644	-63.64	110.00
253	-74.833	-50.00	110.00
254	-83.157	-34.44	110.00
255	-90.000	-17.56	110.00
256	-88.207	10.00	110.00
257	-83.153	17.56	110.00
258	-74.834	34.44	110.00
259	-63.640	50.00	110.00
260	-50.000	63.64	110.00
261	-34.446	74.83	110.00
262	-17.556	83.15	110.00
263	10.000	88.20	110.00
264	17.566	90.00	95.00
265	34.444	83.15	95.00
266	50.000	63.64	95.00
267	63.644	34.44	95.00
268	74.833	17.56	95.00
269	83.157	10.00	95.00
270	90.000	-17.56	95.00
271	88.207	-34.44	95.00
272	83.153	-50.00	95.00
273	74.834	-63.64	95.00
274	63.640	-74.83	95.00
275	50.000	-83.15	95.00
276	34.446	-88.20	95.00
277	17.556	-90.00	95.00
278	10.000	-88.27	95.00
279	-17.566	-83.15	95.00
280	-34.444	-74.83	80.00
281	-50.000	-63.64	80.00
282	-63.644	-50.00	80.00
283	-74.833	-34.44	80.00
284	-83.157	-17.56	80.00

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APPENDIX B

MESH II CONNECTIVITY AND COORDINATES

The connectivity matrix for the 192-element mesh is as follows:

ELEMENT NUMBER	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	3	12	11	10	2	26	28	27	35	37	46	45	44	36
2	1	4	14	13	11	3	26	29	28	35	38	48	47	46	37
3	1	5	18	15	14	4	26	30	29	35	39	50	49	48	38
4	1	6	20	17	16	5	26	31	30	35	40	52	51	50	39
5	1	7	22	19	18	6	26	32	31	35	41	54	53	52	40
6	1	8	24	21	20	7	26	33	32	35	42	56	55	54	41
7	1	9	24	21	20	8	26	34	33	35	43	58	57	56	42
8	1	9	24	21	20	9	26	34	33	35	43	58	57	56	42
9	1	9	24	21	20	9	26	34	33	35	43	58	57	56	42
10	3	37	10	47	46	36	60	62	61	69	71	80	79	78	70
11	3	38	10	47	46	37	60	63	62	69	72	82	81	80	71
12	3	39	10	47	46	38	60	64	63	69	73	84	83	82	72
13	3	40	10	47	46	39	60	65	64	69	74	86	85	84	73
14	3	41	10	47	46	40	60	66	65	69	75	88	87	86	74
15	3	42	10	47	46	41	60	67	66	69	76	90	89	88	75
16	3	43	10	47	46	42	60	68	67	69	77	92	91	90	76
17	3	44	10	47	46	43	60	69	68	69	78	94	93	92	77
18	3	45	10	47	46	44	60	70	69	69	79	96	95	94	78
19	3	46	10	47	46	45	60	71	70	69	80	98	97	96	79
20	3	47	10	47	46	46	60	72	71	69	81	100	99	98	80
21	3	48	10	47	46	47	60	73	72	69	82	101	100	99	81
22	3	49	10	47	46	48	60	74	73	69	83	102	101	100	82
23	3	50	10	47	46	49	60	75	74	69	84	103	102	101	83
24	3	51	10	47	46	50	60	76	75	69	85	104	103	102	84
25	3	52	10	47	46	51	60	77	76	69	86	105	104	103	85
26	3	53	10	47	46	52	60	78	77	69	87	106	105	104	86
27	3	54	10	47	46	53	60	79	78	69	88	107	106	105	87
28	3	55	10	47	46	54	60	80	79	69	89	108	107	106	88
29	3	56	10	47	46	55	60	81	80	69	90	109	108	107	89
30	3	57	10	47	46	56	60	82	81	69	91	110	109	108	90
31	3	58	10	47	46	57	60	83	82	69	92	111	110	109	91
32	3	59	10	47	46	58	60	84	83	69	93	112	111	110	92
33	3	60	10	47	46	59	60	85	84	69	94	113	112	111	93
34	3	61	10	47	46	60	60	86	85	69	95	114	113	112	94
35	3	62	10	47	46	61	60	87	86	69	96	115	114	113	95
36	3	63	10	47	46	62	60	88	87	69	97	116	115	114	96
37	3	64	10	47	46	63	60	89	88	69	98	117	116	115	97
38	3	65	10	47	46	64	60	90	89	69	99	118	117	116	98
39	3	66	10	47	46	65	60	91	90	69	100	119	118	117	99
40	3	67	10	47	46	66	60	92	91	69	101	120	119	118	100
41	3	68	10	47	46	67	60	93	92	69	102	121	120	119	101
42	3	69	10	47	46	68	60	94	93	69	103	122	121	120	102
43	3	70	10	47	46	69	60	95	94	69	104	123	122	121	103
44	3	71	10	47	46	70	60	96	95	69	105	124	123	122	104
45	3	72	10	47	46	71	60	97	96	69	106	125	124	123	105
46	3	73	10	47	46	72	60	98	97	69	107	126	125	124	106
47	3	74	10	47	46	73	60	99	98	69	108	127	126	125	107
48	3	75	10	47	46	74	60	100	99	69	109	128	127	126	108
49	3	76	10	47	46	75	60	101	100	69	110	129	128	127	109
50	3	77	10	47	46	76	60	102	101	69	111	130	129	128	110
51	3	78	10	47	46	77	60	103	102	69	112	131	130	129	111
52	3	79	10	47	46	78	60	104	103	69	113	132	131	130	112
53	3	80	10	47	46	79	60	105	104	69	114	133	132	131	113
54	3	81	10	47	46	80	60	106	105	69	115	134	133	132	114
55	3	82	10	47	46	81	60	107	106	69	116	135	134	133	115
56	3	83	10	47	46	82	60	108	107	69	117	136	135	134	116
57	3	84	10	47	46	83	60	109	108	69	118	137	136	135	117
58	3	85	10	47	46	84	60	110	109	69	119	138	137	136	118
59	3	86	10	47	46	85	60	111	110	69	120	139	138	137	119
60	3	87	10	47	46	86	60	112	111	69	121	140	139	138	120
61	3	88	10	47	46	87	60	113	112	69	122	141	140	139	121
62	3	89	10	47	46	88	60	114	113	69	123	142	141	140	122
63	3	90	10	47	46	89	60	115	114	69	124	143	142	141	123
64	3	91	10	47	46	90	60	116	115	69	125	144	143	142	124
65	3	92	10	47	46	91	60	117	116	69	126	145	144	143	125
66	3	93	10	47	46	92	60	118	117	69	127	146	145	144	126
67	3	94	10	47	46	93	60	119	118	69	128	147	146	145	127
68	3	95	10	47	46	94	60	120	119	69	129	148	147	146	128
69	3	96	10	47	46	95	60	121	120	69	130	149	148	147	129
70	3	97	10	47	46	96	60	122	121	69	131	150	149	148	130
71	3	98	10	47	46	97	60	123	122	69	132	151	150	149	131
72	3	99	10	47	46	98	60	124	123	69	133	152	151	150	132
73	3	100	10	47	46	99	60	125	124	69	134	153	152	151	133
74	3	101	10	47	46	100	60	126	125	69	135	154	153	152	134
75	3	102	10	47	46	101	60	127	126	69	136	155	154	153	135
76	3	103	10	47	46	102	60	128	127	69	137	156	155	154	136
77	3	104	10	47	46	103	60	129	128	69	138	157	156	155	137
78	3	105	10	47	46	104	60	130	129	69	139	158	157	156	138
79	3	106	10	47	46	105	60	131	130	69	140	159	158	157	139
80	3	107	10	47	46	106	60	132	131	69	141	160	159	158	140
81	3	108	10	47	46	107	60	133	132	69	142	161	160	159	141
82	3	109	10	47	46	108	60	134	133	69	143	162	161	160	142
83	3	110	10	47	46	109	60	135	134	69	144	163	162	161	143
84	3	111	10	47	46	110	60	136	135	69	145	164	163	162	144
85	3	112	10	47	46	111	60	137	136	69	146	165	164	163	145
86	3	113	10	47	46	112	60	138	137	69	147	166	165	164	146
87	3	114	10	47	46	113	60	139	138	69	148	167	166	165	147
88	3	115	10	47	46	114	60	140	139	69	149	168	167	166	148
89	3	116	10	47	46	115	60	141	140	69	150	169	168	167	149
90	3	117	10	47	46	116	60	142	141	69	151	170	169	168	150
91	3	118	10	47	46	117	60	143	142	69	152	171	170	169	151
92	3	119	10	47	46	118	60	144	143	69	153	172	171	170	152
93	3	120	10	47	46	119	60	145	144	69	154	173	172	171	153
94	3	121	10	47	46	120	60	146	145	69	155	174	173	172	154
95	3	122	10	47	46	121	60	147	146	69	156	175	174	173	155
96	3	123	10	47	46	122	60	148	147	69	157	176	175	174	156
97	3	124	10	47	46	123	60	149	148	69	158	177	176	175	157
98	3	125	10	47	46	124	60	150	149	69	159	178	177	176	158
99	3	126	10	47	46	125	60	151	150	69	160	179	178	177	159
100	3	127	10												

84	193	5	604	605	191	31	581	580	2	216	557	558
85	193	6	606	607	192	31	582	581	5	217	557	558
86	193	7	608	609	194	58	313	582	54	217	554	553
87	196	1	610	609	195	32	325	583	54	219	559	562
88	196	2	612	611	197	34	325	583	56	220	563	565
89	198	3	614	613	197	33	337	584	56	222	567	566
90	199	4	616	614	198	36	337	584	57	223	569	567
91	201	5	618	617	200	34	348	587	58	225	570	568
92	202	6	620	619	201	38	348	587	57	224	569	572
93	205	7	622	621	203	27	373	588	78	226	571	572
94	205	8	624	623	204	26	373	588	4	229	578	494
95	207	9	626	625	206	62	375	588	95	229	495	496
96	208	1	628	627	207	62	375	588	80	231	497	498
97	208	2	630	629	209	63	375	588	4	232	499	498
98	210	3	632	631	210	63	375	588	80	234	499	500
99	211	4	634	633	212	64	375	588	50	235	501	502
100	213	5	636	635	213	64	375	588	84	237	504	503
101	214	6	638	637	215	65	375	588	50	238	505	506
102	216	7	640	639	216	66	375	588	86	240	507	508
103	217	8	642	641	218	66	375	588	5	241	509	510
104	219	9	644	643	219	67	375	588	11	244	511	512
105	220	1	646	645	221	67	375	588	88	244	513	514
106	222	2	648	647	222	68	375	588	5	246	515	516
107	223	3	650	649	224	68	375	588	90	247	517	518
108	225	4	652	651	225	68	375	588	5	249	519	520
109	226	5	654	653	227	68	375	588	92	250	521	522
110	229	6	656	655	228	69	375	588	5	255	523	524
111	229	7	658	657	230	69	375	588	92	258	523	524
112	231	8	660	659	231	69	375	588	5	253	523	524
113	233	9	662	661	233	69	375	588	92	253	523	524
114	234	1	664	663	233	69	375	588	5	255	523	524
115	235	2	666	665	236	69	375	588	92	256	523	524
116	237	3	668	667	237	69	375	588	5	258	523	524
117	238	4	670	669	238	69	375	588	92	259	523	524
118	240	5	672	671	240	69	375	588	5	254	523	524
119	241	6	674	673	241	69	375	588	92	254	523	524
120	244	7	676	675	244	69	375	588	5	257	523	524
121	249	8	678	677	249	69	375	588	92	257	523	524
122	250	9	680	679	250	69	375	588	5	254	523	524
123	251	1	682	681	251	69	375	588				

180	154	288	413	412	411	287	167	389	388	317	337	365	364	337
181	155	289	414	414	413	288	167	390	389	317	338	365	366	338
182	156	290	415	415	414	290	168	391	390	317	339	367	368	339
183	157	291	416	416	415	291	168	392	391	319	340	369	370	340
184	158	292	417	417	416	293	169	393	392	321	341	371	372	342
185	159	293	418	418	417	294	169	394	393	321	342	373	374	343
186	160	294	419	419	418	296	170	395	394	321	343	375	376	344
187	161	295	420	420	419	296	170	396	395	323	344	377	378	345
188	162	296	421	421	420	297	170	397	396	323	345	377	379	346
189	163	297	422	422	421	299	163	381	396	309	346	309	324	347
190	164	298	423	423	422	299	163		396	309	347	379	380	348
191	165	299	424	424	423						325			
192	166	300	425	425	424									

The coordinates of the 192-element mesh are as follows:

NODE NUMBER	X	Y	Z
1	0.0	0.0	80.00
2	0.0	30.21	80.00
3	21.00	21.00	80.00
4	21.00	-21.00	80.00
5	0.0	-21.00	80.00
6	-21.00	-21.00	80.00
7	-21.00	0.0	80.00
8	-21.00	21.00	80.00
9	0.0	60.00	80.00
10	22.96	42.96	80.00
11	22.96	22.96	80.00
12	55.43	55.43	80.00
13	55.43	22.96	80.00
14	60.00	0.0	80.00
15	55.43	-22.96	80.00
16	42.96	-42.96	80.00
17	22.96	-55.43	80.00
18	0.0	-60.00	80.00
19	-22.96	-55.43	80.00
20	-42.96	-42.96	80.00
21	-55.43	-22.96	80.00
22	-60.00	0.0	80.00
23	-55.43	22.96	80.00
24	-42.96	42.96	80.00
25	-22.96	55.43	80.00
26	0.0	60.00	60.00

27	0.0	60.00	60.00
28	42.43	42.00	60.00
29	60.00	-42.43	60.00
30	42.43	-60.00	60.00
31	0.0	-42.43	60.00
32	-42.43	42.00	60.00
33	-60.00	42.43	60.00
34	-42.43	0.0	60.00
35	0.0	0.0	40.00
36	21.21	30.00	40.00
37	30.00	21.00	40.00
38	21.21	21.21	40.00
39	0.0	-21.21	40.00
40	-21.21	-30.00	40.00
41	-30.00	-21.21	40.00
42	-21.21	21.00	40.00
43	0.0	22.96	40.00
44	22.96	42.43	40.00
45	42.43	55.43	40.00
46	55.43	60.00	40.00
47	60.00	55.43	40.00
48	55.43	42.43	40.00
49	42.43	22.96	40.00
50	22.96	0.0	40.00
51	0.0	-22.96	40.00
52	-22.96	-42.43	40.00
53	-42.43	-55.43	40.00
54	-55.43	-60.00	40.00
55	-60.00	-55.43	40.00
56	-55.43	-42.43	40.00
57	-42.43	-22.96	40.00
58	-22.96	0.0	40.00
59	0.0	22.96	40.00
60	22.96	42.43	40.00
61	42.43	60.00	20.00
62	60.00	42.43	20.00
63	42.43	0.0	20.00
64	0.0	-42.43	20.00
65	-42.43	-60.00	20.00
66	-60.00	-42.43	20.00
67	-42.43	42.00	20.00
68	42.00	42.43	20.00
69	0.0	0.0	0.0
70	0.0	30.00	0.0
71	21.21	21.21	0.0
72	30.00	0.0	0.0
73	21.21	-21.21	0.0
74	0.0	-30.00	0.0

75	-21.21	0.00	0.00
76	-30.00	0.00	0.00
77	-21.21	0.00	0.00
78	0.00	0.00	0.00
79	22.96	0.00	0.00
80	42.43	0.00	0.00
81	55.00	0.00	0.00
82	55.43	0.00	0.00
83	42.43	0.00	0.00
84	2.00	0.00	0.00
85	-22.96	0.00	0.00
86	-42.43	0.00	0.00
87	-55.00	0.00	0.00
88	-55.43	0.00	0.00
89	-42.43	0.00	0.00
90	-2.00	0.00	0.00
91	22.96	0.00	0.00
92	42.43	0.00	0.00
93	55.00	0.00	0.00
94	55.43	0.00	0.00
95	42.43	0.00	0.00
96	2.00	0.00	0.00
97	-22.96	0.00	0.00
98	-42.43	0.00	0.00
99	-55.00	0.00	0.00
100	-55.43	0.00	0.00
101	-42.43	0.00	0.00
102	-2.00	0.00	0.00
103	22.96	0.00	0.00
104	42.43	0.00	0.00
105	55.00	0.00	0.00
106	55.43	0.00	0.00
107	42.43	0.00	0.00
108	2.00	0.00	0.00
109	-22.96	0.00	0.00
110	-42.43	0.00	0.00
111	-55.00	0.00	0.00
112	-55.43	0.00	0.00
113	-42.43	0.00	0.00
114	-2.00	0.00	0.00
115	22.96	0.00	0.00
116	42.43	0.00	0.00
117	55.00	0.00	0.00
118	55.43	0.00	0.00
119	42.43	0.00	0.00
120	2.00	0.00	0.00
121	-22.96	0.00	0.00
122	-42.43	0.00	0.00

123	-5.43	-2.96	-40.00
124	-55.43	22.96	-40.00
125	-55.43	42.43	-40.00
126	-42.96	55.43	-40.00
127	-22.96	0.00	-60.00
128	0.00	62.43	-60.00
129	42.43	4.00	-60.00
130	60.00	0.43	-60.00
131	42.43	-42.43	-60.00
132	0.43	-60.00	-60.00
133	-42.43	-42.43	-60.00
134	-60.00	0.43	-60.00
135	-42.43	42.43	-60.00
136	0.00	30.00	-80.00
137	0.00	21.21	-80.00
138	21.21	0.00	-80.00
139	30.00	21.21	-80.00
140	21.21	-21.21	-80.00
141	0.00	-21.21	-80.00
142	-21.21	0.00	-80.00
143	-30.00	21.21	-80.00
144	-21.21	21.21	-80.00
145	0.00	55.43	-80.00
146	22.43	42.96	-80.00
147	42.96	22.00	-80.00
148	55.43	22.96	-80.00
149	55.43	-22.96	-80.00
150	42.96	-55.43	-80.00
151	22.96	-42.96	-80.00
152	0.00	-22.96	-80.00
153	-22.96	0.00	-80.00
154	-42.96	22.96	-80.00
155	-55.43	42.96	-80.00
156	-42.96	55.43	-80.00
157	-22.96	0.00	-80.00
158	0.00	42.96	-80.00
159	42.96	55.43	-80.00
160	55.43	42.96	-80.00
161	42.96	0.00	-95.00
162	0.00	42.43	-95.00
163	42.43	0.00	-95.00
164	50.00	42.43	-95.00
165	42.43	0.00	-95.00
166	0.00	-42.43	-95.00
167	-42.43	-60.00	-95.00
168	-60.00	-42.43	-95.00
169	-42.43	0.00	-95.00
170	-42.43	42.43	-95.00

171	0.0	0.0	95.00
172	2.43	60.00	95.00
173	60.00	42.03	95.00
174	42.43	0.0	95.00
175	42.43	-42.43	95.00
176	0.0	-60.00	95.00
177	-42.42	-42.03	95.00
178	-42.43	0.0	95.00
179	-42.43	42.43	95.00
180	0.0	75.00	80.00
181	16.80	71.90	80.00
182	38.10	62.03	80.00
183	53.03	53.03	80.00
184	62.80	38.60	80.00
185	71.30	17.00	80.00
186	75.00	0.0	80.00
187	71.30	-17.00	80.00
188	52.80	-38.60	80.00
189	53.03	-53.03	80.00
190	38.10	-62.90	80.00
191	17.10	-71.80	80.00
192	17.0	-75.00	80.00
193	0.0	-71.80	80.00
194	-16.90	-62.80	80.00
195	-38.50	-53.03	80.00
196	-53.03	-38.60	80.00
197	-62.70	-17.10	80.00
198	-71.30	0.0	80.00
199	-75.00	17.10	80.00
200	-62.90	38.30	80.00
201	-53.03	53.03	80.00
202	-38.60	62.80	80.00
203	-17.0	71.30	80.00
204	0.0	75.00	40.00
205	16.80	71.60	40.00
206	38.10	62.90	40.00
207	53.03	53.03	40.00
208	62.80	38.60	40.00
209	71.30	17.00	40.00
210	75.00	0.0	40.00
211	71.30	-17.00	40.00
212	52.80	-38.60	40.00
213	53.03	-53.03	40.00
214	38.10	-62.90	40.00
215	17.10	-71.80	40.00
216	17.0	-75.00	40.00
217	-16.90	-71.80	40.00
218	-38.50	-62.80	40.00

219	53.03	-53.03	40.00
220	-62.70	-38.60	40.00
221	-71.30	-17.10	40.00
222	-75.00	0.00	40.00
223	-71.30	17.10	40.00
224	-62.90	38.30	40.00
225	-53.03	53.03	40.00
226	-38.60	62.80	40.00
227	-17.30	71.30	40.00
228	-17.30	75.00	0.00
229	16.80	71.60	0.00
230	38.10	62.90	0.00
231	53.03	53.03	0.00
232	62.80	38.60	0.00
233	71.30	17.00	0.00
234	75.00	0.00	0.00
235	71.30	17.00	0.00
236	52.80	-17.60	0.00
237	53.03	-53.03	0.00
238	38.10	-62.90	0.00
239	17.10	-71.80	0.00
240	17.10	-75.00	0.00
241	-16.90	-71.80	0.00
242	-38.50	-62.80	0.00
243	-53.03	-53.03	0.00
244	-52.70	-38.60	0.00
245	-71.30	-17.10	0.00
246	-75.00	0.00	0.00
247	-71.30	17.10	0.00
248	-62.90	38.30	0.00
249	-53.03	53.03	0.00
250	-38.60	62.80	0.00
251	-17.30	71.30	0.00
252	-17.30	75.00	0.00
253	16.80	71.60	-40.00
254	38.10	62.90	-40.00
255	53.03	53.03	-40.00
256	62.80	38.60	-40.00
257	71.30	17.00	-40.00
258	75.00	0.00	-40.00
259	71.30	17.00	-40.00
260	62.80	-17.60	-40.00
261	53.03	-53.03	-40.00
262	38.10	-62.90	-40.00
263	17.10	-71.80	-40.00
264	17.10	-75.00	-40.00
265	-16.90	-71.80	-40.00
266	-38.50	-62.80	-40.00

267	-53.03	-53.00	-40.00
268	-62.70	-38.60	-40.00
269	-71.30	-17.10	-40.00
270	-75.00	17.10	-40.00
271	-71.30	38.30	-40.00
272	-62.90	53.03	-40.00
273	-53.03	62.80	-40.00
274	-38.60	71.30	-40.00
275	-17.30	75.00	-80.00
276	-10.00	71.60	-80.00
277	16.80	62.90	-80.00
278	38.10	53.03	-80.00
279	53.03	38.60	-80.00
280	62.80	17.00	-80.00
281	71.30	17.00	-80.00
282	75.00	17.00	-80.00
283	71.30	-17.00	-80.00
284	62.80	-38.60	-80.00
285	53.03	-53.03	-80.00
286	38.10	-62.90	-80.00
287	17.00	-71.80	-80.00
288	16.90	-75.00	-80.00
289	-38.50	-71.80	-80.00
290	-53.03	-62.80	-80.00
291	-62.70	-53.03	-80.00
292	-71.30	-38.60	-80.00
293	-75.00	-17.10	-80.00
294	-71.30	17.00	-80.00
295	-62.90	17.10	-80.00
296	-53.03	38.30	-80.00
297	-38.60	53.03	-80.00
298	-17.30	62.80	-80.00
299	-10.00	71.30	-80.00
300	0.00	30.00	-110.00
301	21.21	21.21	-110.00
302	30.00	21.21	-110.00
303	21.21	21.00	-110.00
304	20.00	-21.21	-110.00
305	-21.21	-21.21	-110.00
306	-21.21	-21.00	-110.00
307	-30.00	21.21	-110.00
308	-21.21	21.00	-110.00
309	22.96	60.43	-110.00
310	42.43	55.43	-110.00
311	55.43	42.96	-110.00
312	60.43	22.00	-110.00
313	55.43	0.96	-110.00
314	60.43	-22.96	-110.00

[illegible]

411	34.44	-83.15	-80.00
412	17.56	-88.27	-80.00
413	17.00	-90.07	-80.00
414	-17.56	-88.15	-80.00
415	-34.44	-74.83	-80.00
416	-50.00	-63.64	-80.00
417	-63.64	-74.83	-80.00
418	-74.83	-50.00	-80.00
419	-83.15	-34.44	-80.00
420	-88.27	-17.56	-80.00
421	-90.00	17.00	-80.00
422	-88.27	17.56	-80.00
423	-83.15	34.44	-80.00
424	-74.83	50.00	-80.00
425	-63.64	63.64	-80.00
426	-50.00	74.83	-80.00
427	-34.44	83.15	-80.00
428	-17.56	88.27	-80.00
429	-17.00	90.00	-60.00
430	34.44	83.15	-60.00
431	63.64	63.64	-60.00
432	83.15	34.44	-60.00
433	90.00	0.00	-60.00
434	83.15	-34.44	-60.00
435	63.64	-63.64	-60.00
436	34.44	-90.00	-60.00
437	0.00	-83.15	-60.00
438	-34.44	-63.64	-60.00
439	-63.64	-34.44	-60.00
440	-83.15	34.44	-60.00
441	-90.00	63.64	-60.00
442	-83.15	83.15	-60.00
443	-63.64	90.00	-40.00
444	-34.44	88.27	-40.00
445	0.00	83.15	-40.00
446	17.56	74.83	-40.00
447	34.44	50.00	-40.00
448	50.00	34.44	-40.00
449	63.64	17.56	-40.00
450	74.83	17.00	-40.00
451	83.15	0.00	-40.00
452	88.27	-17.56	-40.00
453	90.00	-34.44	-40.00
454	88.27	-50.00	-40.00
455	83.15	-63.64	-40.00
456	74.83	-74.83	-40.00
457	63.64		
458	50.00		

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 50.00
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 -17.56
 -34.44
 -50.00
 -63.64
 -74.83

[illegible]

507	34.44	-83.15	0.0
508	17.56	-88.27	0.0
509	17.56	-90.27	0.0
510	-17.56	-88.27	0.0
511	-34.44	-83.15	0.0
512	-50.00	-74.83	0.0
513	-63.64	-63.64	0.0
514	-74.83	-50.00	0.0
515	-83.15	-34.44	0.0
516	-88.27	-17.56	0.0
517	-90.00	17.56	0.0
518	-88.27	34.44	0.0
519	-74.83	50.00	0.0
520	-63.64	63.64	0.0
521	-50.00	74.83	0.0
522	-34.44	83.15	0.0
523	-17.56	88.27	0.0
524	17.56	90.00	0.0
525	34.44	83.15	20.00
526	63.64	63.64	20.00
527	83.15	34.44	20.00
528	90.00	0.0	20.00
529	83.15	-34.44	20.00
530	63.64	-63.64	20.00
531	34.44	-83.15	20.00
532	0.0	-90.00	20.00
533	-34.44	-83.15	20.00
534	-63.64	-63.64	20.00
535	-83.15	-34.44	20.00
536	-90.00	17.56	20.00
537	-88.27	34.44	20.00
538	-74.83	50.00	20.00
539	-63.64	63.64	20.00
540	-50.00	74.83	40.00
541	-34.44	83.15	40.00
542	17.56	88.27	40.00
543	34.44	90.00	40.00
544	50.00	83.15	40.00
545	63.64	74.83	40.00
546	74.83	63.64	40.00
547	83.15	50.00	40.00
548	88.27	34.44	40.00
549	90.00	17.56	40.00
550	88.27	-17.56	40.00
551	74.83	-34.44	40.00
552	63.64	-50.00	40.00
553	50.00	-63.64	40.00
554	34.44	-74.83	40.00

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KK=CONN(I,K)
IF(KK.GT.183) GO TO 80
JA(JJ)=JA(JJ)+I
JAA=JA(JJ)
DO 70 L=2,JAA
JJL=MAME(JJ,L)
IF(JJL.EQ.KK) JA(JJ)=JA(JJ)-1
IF(JJL.EQ.KK) GO TO 80
IF(JJL.EQ.0) MAME(JJ,JAA)=KK
CONTINUE
70 CONTINUE
80 CONTINUE
90 CONTINUE
100 CONTINUE
C
JB(1)=1
JC=0
DO 200 I=1,183
JN=JA(I)
JB(I+1)=JB(I)+JA(I)
JC=JC+JA(I)
CONTINUE
200 WRITE(6,215) JC
DO 250 I=1,183
JAA=JA(I)
JBL=JB(I)
DO 240 J=1,JAA
JJ=JBL+J-1
NAME(JJ)=MAME(I,J)
CONTINUE
240 CONTINUE
250 WRITE(6,204)
WRITE(6,205) (JA(I),I=1,183)
WRITE(6,207)
WRITE(6,205) (JB(I),I=1,183)
WRITE(6,208)
WRITE(6,205) (NAME(I),I=1,JC)
STOP
END

```



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C-----
C  THIS IS THE MAIN PROGRAM WHICH CALCULATES THE BIGG AND BIGGG
C  SYSTEM MATRICES BY USING SUBROUTINES FLOWIE AND TANYA. THE SYSTEM
C  MATRICES ARE THEN PUT ON TAPE NUMBER NPS182.
C-----
      REAL*8  G,GG,BIGG,BIGGG
      REAL*8  WL,WS,S,CL1,CL2,CL3,FN,DL1,DL2,DL3,DS,DETJ
      INTEGER*2 NAME,JA,JB,NNZ,JC,NUMNP,NUMEL,NELDOF,CONN
      COMMON/ GTRY2/NAME(4615),JA(183),JB(183),NNZ,JC,NUMNP,NUMEL,NELDOF,
1      CONN(128,15)
      COMMON/ GTRY1/ X(505),P(505),Z(505)
      COMMON/ XOCAL/ XX(15),YY(15),ZZ(15),PPSI(15)
      COMMON/ GMAT/ G(15,15)
      COMMON/ GGMAT/ GG(15,15)
      COMMON/ SYSMT1/ BIGG(4615)
      COMMON/ SYSMT2/ BIGGG(4615)
      COMMON/ GAJSS/ WL(7),WS(5),S(5),CL1(7),CL2(7),CL3(7),FN(15,21),DL1(1
15,21),DL2(15,21),DL3(15,21),DS(15,21),DETJ(21)
150  FORMAT(16I5)
160  FORMAT(25I5)
211  FORMAT(8E16.8)
609  FORMAT(12F10.4)
702  FORMAT(20I6)
800  FORMAT(10F8.2)
952  FORMAT(20I4)
      NNZ=183
      NUMEL=128
      NUMNP=505
      JC=4615
      NELDOF=15
455  DO 455 I=1,NUMEL
      READ(5,150) I,(CONN(I,J),J=1,NELDOF)
      CONTINUE
      READ(5,952) (JA(I),I=1,NNZ)
      READ(5,952) (JB(I),I=1,NNZ)
      READ(5,952) (NAME(I),I=1,JC)
      READ(5,800) (X(I),I=1,NUMNP)
      READ(5,800) (P(I),I=1,NUMNP)
      READ(5,800) (Z(I),I=1,NUMNP)
      DO 465 I=1,JC
      BIGG(I)=0.0
      BIGGG(I)=0.0
465  CONTINUE
      CALL TANYA
      CALL FLOWIE
      STOP

```

M-010
M-015

M-060

M-100

M-120
M-125
M-130

M-160
M-140
M-145

M-355
M-360
M-365
M-370
M-405
M-415
M-615

END

M-620

THIS SUBROUTINE EVALUATES THE GG ELEMENT MATRIX USING 20 POINTS OF
INTEGRATION. THE GG ELEMENT MATRIX IS THEN INSERTED INTO THE BIGGG
SYSTEM MATRIX.

SUBROUTINE TANYA

REAL*8 GG,BIGGG

REAL*8 WL,WS,S,CL1,CL2,CL3,FN,DL1,DL2,DL3,DS,DETJ

INTEGER*2 NAME,JA,JB,NNZ,JC,NJMNP,NUMEL,NELDOF,CONN

COMMON/GTRY2/NAME(8199),JA(299),JB(299),NNZ,JC,NUMNP,NUMEL,NELDOF,

1CONN(192,15)

COMMON/GTRY1/X(717),P(717),Z(717)

COMMON/XOCAL/KX(15),YY(15),ZZ(15),PPSI(15)

COMMON/GGMAT/GG(15,15)

COMMON/SYSMT2/BIGGG(8199)

COMMON/GAUSS/WL(7),WS(5),S(5),CL1(7),CL2(7),CL3(7),FN(15,21),DL1(1

15,21),DL2(15,21),DL3(15,21),DS(15,21),DETJ(21)

WS(1)=0.236926885056189

WS(2)=0.478628670499366

WS(3)=0.56888888888889

WS(4)=0.478628670499366

WS(5)=0.236926885056189

S(1)=0.906179845938664

S(2)=0.538469310105683

S(3)=0.0

S(4)=0.538469310105683

S(5)=0.906179845938664

WL(1)=0.281250

WL(2)=0.260416666666667

WL(3)=0.260416666666667

WL(4)=0.260416666666667

WL(5)=0.260416666666667

CL1(1)=0.333333333333333

CL1(2)=0.60

CL1(3)=0.20

CL1(4)=0.20

CL2(1)=0.333333333333333

CL2(2)=0.20

CL2(3)=0.60

CL2(4)=0.20

CL3(1)=0.333333333333333

CL3(2)=0.20

CL3(3)=0.60

CL3(4)=0.20

DO 80 K=1,5

DO 90 M=1,4

T-015


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IF(K.EQ.1) GO TO 10
IF(K.EQ.2) GO TO 20
IF(K.EQ.3) GO TO 30
IF(K.EQ.4) GO TO 40
IF(K.EQ.5) GO TO 50
10 N=M
GO TO 60
20 N=M+4
GO TO 60
30 N=M+8
GO TO 60
40 N=M+12
GO TO 60
50 N=M+16
GO TO 60
60 CONTINUE
FN(1,V)= 0.50*CL1(M)*((2.0*CL1(M)-1.0)*(1.0+S(K))-(1.0-S(K)**2))
FN(2,V)= 2.0*CL1(M)*CL2(M)*(1.0+S(K))
FN(3,V)= 0.50*CL2(M)*((2.0*CL2(M)-1.0)*(1.0+S(K))-(1.0-S(K)**2))
FN(4,V)= 2.0*CL2(M)*CL3(M)*(1.0+S(K))
FN(5,V)= 0.50*CL3(M)*((2.0*CL3(M)-1.0)*(1.0+S(K))-(1.0-S(K)**2))
FN(6,V)= 2.0*CL3(M)*CL1(M)*(1.0+S(K))
FN(7,V)= CL1(M)*((1.0-S(K)**2))
FN(8,V)= CL2(M)*((1.0-S(K)**2))
FN(9,V)= CL3(M)*((1.0-S(K)**2))
FN(10,V)= 0.50*CL1(M)*((2.0*CL1(M)-1.0)*(1.0-S(K))-(1.0-S(K)**2))
FN(11,V)= 2.0*CL1(M)*CL2(M)*(1.0-S(K))
FN(12,V)= 0.50*CL2(M)*((2.0*CL2(M)-1.0)*(1.0-S(K))-(1.0-S(K)**2))
FN(13,V)= 2.0*CL2(M)*CL3(M)*(1.0-S(K))
FN(14,V)= 0.50*CL3(M)*((2.0*CL3(M)-1.0)*(1.0-S(K))-(1.0-S(K)**2))
FN(15,V)= 2.0*CL3(M)*CL1(M)*(1.0-S(K))
DL1(1,V)= 0.50*CL1(M)*CL1(M)-1.0)
DL1(2,V)= 2.0*CL2(M)*(1.0+S(K))
DL1(3,V)= 0.0
DL1(4,V)= 0.0
DL1(5,V)= 0.0
DL1(6,V)= 2.0*CL3(M)*(1.0+S(K))
DL1(7,V)= (1.0-S(K)**2)
DL1(8,V)= 0.0
DL1(9,V)= 0.0
DL1(10,V)= 0.50*CL1(M)*((4.0*CL1(M)-1.0)-(1.0-S(K)**2))
DL1(11,V)= 2.0*CL2(M)*(1.0-S(K))
DL1(12,V)= 0.0
DL1(13,V)= 0.0
DL1(14,V)= 0.0
DL1(15,V)= 2.0*CL3(M)*(1.0-S(K))
DL2(1,V)= 0.0
DL2(2,V)= 2.0*CL1(M)*(1.0+S(K))
DL2(3,V)= 0.50*CL1(M)*CL2(M)-1.0)-(1.0-S(K)**2))

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DL2(4,N)=2.0*CL3(M)*(1.0+S(K))
DL2(5,N)=0.0
DL2(6,N)=0.0
DL2(7,N)=0.0
DL2(8,N)=(1.0-S(K)**2)
DL2(9,N)=0.0
DL2(10,N)=0.0
DL2(11,N)=2.0*CL1(M)*(1.0-S(K))
DL2(12,N)=0.50*((1.0-S(K))*(4.0*CL2(M)-1.0)-(1.0-S(K)**2))
DL2(13,N)=2.0*CL3(M)*(1.0-S(K))
DL2(14,N)=0.0
DL2(15,N)=0.0
DL3(1,N)=0.0
DL3(2,N)=0.0
DL3(3,N)=0.0
DL3(4,N)=2.0*CL2(M)*(1.0+S(K))
DL3(5,N)=0.50*((1.0+S(K))*(4.0*CL3(M)-1.0)-(1.0-S(K)**2))
DL3(6,N)=2.0*CL1(M)*(1.0+S(K))
DL3(7,N)=0.0
DL3(8,N)=0.0
DL3(9,N)=(1.0-S(K)**2)
DL3(10,N)=0.0
DL3(11,N)=0.0
DL3(12,N)=0.0
DL3(13,N)=2.0*CL2(M)*(1.0-S(K))
DL3(14,N)=0.50*((1.0-S(K))*(4.0*CL3(M)-1.0)-(1.0-S(K)**2))
DL3(15,N)=2.0*CL1(M)*(1.0-S(K))
DS(1,V)=0.50*(2.0*CL1(M)**2-CL1(M))+CL1(M)*S(K)
DS(2,N)=2.0*CL1(M)*CL2(M)
DS(3,N)=0.50*(2.0*CL2(M)**2-CL2(M))+CL2(M)*S(K)
DS(4,V)=2.0*CL2(M)*CL3(M)
DS(5,N)=0.50*(2.0*CL3(M)**2-CL3(M))+CL3(M)*S(K)
DS(6,V)=2.0*CL1(M)*CL3(M)
DS(7,N)=-2.0*CL1(M)*S(K)
DS(8,V)=-2.0*CL2(M)*S(K)
DS(9,N)=-2.0*CL3(M)*S(K)
DS(10,N)=-50*(2.0*CL1(M)**2-CL1(M))+CL1(M)*S(K)
DS(11,N)=-50*(2.0*CL1(M)*CL2(M))
DS(12,N)=-50*(2.0*CL2(M)**2-CL2(M))+CL2(M)*S(K)
DS(13,V)=-50*(2.0*CL2(M)*CL3(M))
DS(14,N)=-50*(2.0*CL3(M)**2-CL3(M))+CL3(M)*S(K)
DS(15,V)=-2.0*CL1(M)*CL3(M)
90 CONTINUE
80
DO 510 L=1,NUMEL
DO 475 I=1,NELDOF
DO 470 J=1,NELDOF
GG(I,J)=0.0

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470 CONTINUE
475 CONTINUE
DO 480 JJ=1,NELDOF
NN=CONN(L,JJ)
XX(JJ)=X(NN)
YY(JJ)=P(NN)
ZZ(JJ)=Z(NN)
480 CONTINUE
A5=2.0/(ZZ(1)-ZZ(15))
DETJJ=0.0
DO 200 K=1,5
DETJ1=0.0
DO 210 M=1,4
IF(K.EQ.1) GO TO 11
IF(K.EQ.2) GO TO 21
IF(K.EQ.3) GO TO 31
IF(K.EQ.4) GO TO 41
IF(K.EQ.5) GO TO 51
11 N=M
GO TO 61
21 N=M+4
GO TO 61
31 N=M+8
GO TO 61
41 N=M+12
GO TO 61
51 N=M+16
GO TO 61
61 CONTINUE
T1=DL1(1,N)*XX(1)+DL1(2,N)*XX(2)-DL3(4,N)*XX(4)-DL3(5,N)*XX(5)
T2=DL1(7,N)*XX(7)-DL3(9,N)*XX(9)+DL1(10,N)*XX(10)+DL1(11,N)*XX(11)
T3=(DL1(6,N))*XX(6)-DL3(13,N)*XX(13)-DL3(14,N)*XX(14)
T4=(DL1(15,N))-DL3(15,N)*XX(15)
DJ11=T1+T2+T3+T4
D1=DL1(1,N)*YY(1)+DL1(2,N)*YY(2)-DL3(4,N)*YY(4)-DL3(5,N)*YY(5)
D2=DL1(7,N)*YY(7)-DL3(9,N)*YY(9)+DL1(10,N)*YY(10)+DL1(11,N)*YY(11)
D3=(DL1(6,N))*YY(6)-DL3(13,N)*YY(13)-DL3(14,N)*YY(14)
D4=(DL1(15,N))-DL3(15,N)*YY(15)
DJ12=D1+D2+D3+D4
T5=DL2(2,N)*XX(2)+DL2(3,N)*XX(3)-DL3(5,N)*XX(5)-DL3(6,N)*XX(6)
T6=DL2(8,N)*XX(8)-DL3(9,N)*XX(9)+DL2(4,N)*XX(4)
T7=DL2(11,N)*XX(11)+DL2(12,N)*XX(12)+(DL2(13,N)-DL3(13,N))*XX(13)
T8=-DL3(14,N)*XX(14)-DL3(15,N)*XX(15)
DJ21=T5+T6+T7+T8
D5=DL2(2,N)*YY(2)+DL2(3,N)*YY(3)-DL3(5,N)*YY(5)-DL3(6,N)*YY(6)
D6=DL2(8,N)*YY(8)-DL3(9,N)*YY(9)+DL2(4,N)*YY(4)
D7=DL2(11,N)*YY(11)+DL2(12,N)*YY(12)+(DL2(13,N)-DL3(13,N))*YY(13)
D8=-DL3(14,N)*YY(14)-DL3(15,N)*YY(15)
DJ22=D5+D6+D7+D8

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DETJ(N)=DJ11*DJ22-DJ12*DJ21
DETJ1=DETJ1+D=J(N)*WL(M)
CONTINUE
210 DETJJ=DETJJ+WS(K)*DETJ1
CONTINUE
200 DO 600 J=1,15
DO 610 I=1,15
FEG=0.0
DO 660 K=1,5
FEG=0.0
DO 670 M=1,4
IF(K.EQ.1) GO TO 12
IF(K.EQ.2) GO TO 22
IF(K.EQ.3) GO TO 32
IF(K.EQ.4) GO TO 42
IF(K.EQ.5) GO TO 52
12 N=M
GO TO 62
22 N=M+4
GO TO 62
32 N=M+8
GO TO 62
42 N=M+12
GO TO 62
52 N=M+16
GO TO 62
62 CONTINUE
DXDL5=DL1(1,N)*XX(1)+(DL1(2,N)-DL2(2,N))*XX(2)-DL2(3,N)*XX(3)
DXDL5=-{(DL2(4,N)+DL3(4,N))*XX(4)-DL3(5,N)*XX(5)}
DXDL7=(DL1(6,N)-DL3(6,N))*XX(6)+DL1(7,N)*XX(7)-DL2(8,N)*XX(8)
DXDL9=-DL3(9,N)*XX(9)+DL1(10,N)*XX(10)+(DL1(11,N)-DL2(11,N))*XX(11)
1)
DXDL9=-DL2(12,N)*XX(12)-(DL2(13,N)+DL3(13,N))*XX(13)-DL3(14,N)*XX(14)
1)
DXDL1=(DL1(15,N)-DL3(15,N))*XX(15)
DXDL1=DXDL5+DXDL6+DXDL7+DXDL8+DXDL9
DXDL5=DL1(1,N)*YY(1)+(DL1(2,N)-DL2(2,N))*YY(2)-DL2(3,N)*YY(3)
DXDL6=-{(DL2(4,N)+DL3(4,N))*YY(4)-DL3(5,N)*YY(5)}
DXDL7=(DL1(6,N)-DL3(6,N))*YY(6)+DL1(7,N)*YY(7)-DL2(8,N)*YY(8)
DXDL8=-DL3(9,N)*YY(9)+DL1(10,N)*YY(10)+(DL1(11,N)-DL2(11,N))*YY(11)
1)
DXDL9=-DL2(12,N)*YY(12)-(DL2(13,N)+DL3(13,N))*YY(13)-DL3(14,N)*YY(14)
1)
DXDL1=(DL1(15,N)-DL3(15,N))*YY(15)
DXDL1=DXDL5+DXDL6+DXDL7+DXDL8+DXDL9
DXDL55=-DL1(1,N)*XX(1)+(DL2(2,N)-DL1(2,N))*XX(2)+DL2(3,N)*XX(3)
DXDL56=(DL2(4,N)-DL3(4,N))*XX(4)-DL3(5,N)*XX(5)-DL1(7,N)*XX(7)
DXDL77=-{(DL1(5,N)+DL3(6,N))*XX(6)+DL2(8,N)*XX(8)-DL3(9,N)*XX(9)}
DXDL88=-DL1(10,N)*XX(10)+(DL2(11,N)-DL1(11,N))*XX(11)
DXDL99=DL2(12,N)*XX(12)+(DL2(13,N)-DL3(13,N))*XX(13)-DL3(14,N)*XX(14)
1)
DXDL14=-DL1(15,N)+DL3(15,N))*XX(15)

```



```

DXDL2=DXDL55+DXDL66+DXDL77+DXDL88+DXDL99
DYDL55=-DL1(1,N)*YY(1)+(DL2(2,N)-DL1(2,N))*YY(2)+DL2(3,N)*YY(3)
DYDL66=(DL2(4,N)-DL3(4,N))*YY(4)-DL3(5,N)*YY(5)-DL1(7,N)*YY(7)
DYDL77=-DL1(6,N)+DL3(6,N))*YY(6)+DL2(8,N)*YY(8)-DL3(9,N)*YY(9)
DYDL88=-DL1(10,N)*YY(10)+(DL2(11,N)-DL1(11,N))*YY(11)
DYDL99=DL2(12,N)*YY(12)+(DL2(13,N)-DL3(13,N))*YY(13)-DL3(14,N)*YY(
114)-(DL1(15,N)+DL3(15,N))*YY(15)
DYDL2=DYDL55+DYDL66+DYDL77+DYDL88+DYDL99
DL1DY=1.0/DYDL1
DL2DX=1.0/DXDL2
DL1DX=1.0/DXDL1
DL2DY=1.0/DYDL2
BB11=(DL1DX**2)+(DL1DY**2)
BB12=DL1DX*DL2DX+DL1DY*DL2DY
BB22=(DL2DX**2)+(DL2DY**2)
H1=BB11*((DL1(J,N)-DL3(J,N))*DL1(1,N)-DL3(1,N))
H2=BB12*((DL2(J,N)-DL3(J,N))*DL1(1,N)-DL3(1,N))
H3=BB12*((DL1(J,N)-DL3(J,N))*DL2(1,N)-DL3(1,N))
H4=BB22*((DL2(J,N)-DL3(J,N))*DL2(1,N)-DL3(1,N))
H5=(A5**2)*DS(I,N)*DS(I,N)
H6=(H1+H2+H3+H4+H5)*DETJ(N)
FFG=FFG+WL(M)*H6
670 CONTINUE
FFG=WS(K)*FFG+FFG
660 CONTINUE
GG(J,I)=FFG/A5
610 CONTINUE
600 CONTINUE
*** INSERT ELEMENT MATRIX INTO SYSTEM MATRIX *****
DO 505 K=1,NELDOF
KK=CONN(L,K)
IF(KK.GT.NNZ) GO TO 505
KKK=JA(KK)
LLL=JB(KK)-1
DO 500 I=1,NELDOF
II=CONN(L,I)
IF(II.GT.NNZ) GO TO 500
DO 490 M=1,KKK
MM=LLL+M
KKM=NAME(MM)
IF(II.EQ.KKM) GO TO 495
490 CONTINUE
495 CONTINUE
BIGGG(MM)=BIGGG(MM)+GG(K,I)
500 CONTINUE
505 CONTINUE
510 RETURN

```


END

THIS SUBROUTINE EVALUATES THE G ELEMENT MATRIX USING 21 POINTS OF
INTEGRATION. THE G ELEMENT MATRIX IS THEN INSERTED INTO THE BIGG
SYSTEM MATRIX.

SUBROUTINE FLOWIE

```

REAL*8 G,BIGG
REAL*8 WL,WS,S,CL1,CL2,CL3,FN,DL1,DL2,DL3,DS,DETJ
INTEGER*2 NAME,JA,JB,NNZ,JC,NJMNP,NUMEL,NELDOF,CONN
COMMON/GTRY2/NAME(8199),JA(299),JB(299),NNZ,JC,NUMNP,NUMEL,NELDOF,
1 CONN(192,15)
COMMON/GIRYL/X(717),P(717),Z(717)
COMMON/XOCAL/XX(15),YY(15),ZZ(15),PPSI(15)
COMMON/GMAT/G(15,15)
COMMON/SYSTL/BIGG(8199)
COMMON/GAUSS/WL(7),WS(5),S(5),CL1(7),CL2(7),CL3(7),FN(15,21),DL1(1
1 5,21),DL2(15,21),DL3(15,21),DS(15,21),DETJ(21)
WS(1)=0.5555555555555556
WS(2)=0.8888888888888889
WS(3)=0.5555555555555556
S(1)=0.774596669241483
S(2)=0.0
S(3)=-.774596669241483
WL(1)=0.112500
WL(2)=0.06619707500
WL(3)=0.06619707500
WL(4)=0.06619707500
WL(5)=0.0629695900
WL(6)=0.0629695900
WL(7)=0.0629695900
CL1(1)=0.333333333333333
CL1(2)=0.059615870
CL1(3)=0.470142060
CL1(4)=0.470142060
CL1(5)=0.797426990
CL1(6)=0.101286510
CL1(7)=0.101286510
CL2(1)=0.333333333333333
CL2(2)=0.470142060
CL2(3)=0.059615870
CL2(4)=0.470142060
CL2(5)=0.101286510
CL2(6)=0.797426990
CL2(7)=0.101286510
CL3(1)=0.333333333333333

```



```

CL3(2)=0.470142060
CL3(3)=0.470142060
CL3(4)=0.059615870
CL3(5)=0.101286510
CL3(6)=0.101286510
CL3(7)=0.797426990
DO 10 K=1,3
DO 20 M=1,7
C THE 3 X 7 MATRICES ARE BEING CONVERTED TO 1 X 21 VECTORS
30 N=M
IF(K-2)30,40,50
40 N=M+7
50 N=M+14
60 CONTINUE
FN(1,N)=0.50*CL1(M)*((2.0*CL1(M)-1.0)*(1.0+S(K))-(1.0-S(K)**2))
FN(2,N)=2.0*CL1(M)*CL2(M)*(1.0+S(K))
FN(3,N)=0.50*CL2(M)*((2.0*CL2(M)-1.0)*(1.0+S(K))-(1.0-S(K)**2))
FN(4,N)=2.0*CL2(M)*CL3(M)*(1.0+S(K))
FN(5,N)=0.50*CL3(M)*((2.0*CL3(M)-1.0)*(1.0+S(K))-(1.0-S(K)**2))
FN(6,N)=2.0*CL1(M)*CL3(M)*(1.0+S(K))
FN(7,N)=CL1(M)*(1.0-S(K)**2)
FN(8,N)=CL2(M)*(1.0-S(K)**2)
FN(9,N)=CL3(M)*((2.0*CL1(M)-1.0)*(1.0-S(K)**2))
FN(10,N)=0.50*CL1(M)*((2.0*CL1(M)-1.0)*(1.0-S(K)**2))
FN(11,N)=2.0*CL1(M)*CL2(M)*(1.0-S(K))
FN(12,N)=0.50*CL2(M)*((2.0*CL2(M)-1.0)*(1.0-S(K)**2))
FN(13,N)=2.0*CL2(M)*CL3(M)*(1.0-S(K))
FN(14,N)=0.50*CL3(M)*((2.0*CL3(M)-1.0)*(1.0-S(K)**2))
FN(15,N)=2.0*CL1(M)*CL3(M)*(1.0-S(K))
DL1(1,N)=0.50*((1.0+S(K))*(4.0*CL1(M)-1.0)-(1.0-S(K)**2))
DL1(2,N)=2.0*CL2(M)*(1.0+S(K))
DL1(3,N)=0.0
DL1(4,N)=0.0
DL1(5,N)=0.0
DL1(6,N)=2.0*CL3(M)*(1.0+S(K))
DL1(7,N)=(1.0-S(K)**2)
DL1(8,N)=0.0
DL1(9,N)=0.0
DL1(10,N)=0.50*((1.0-S(K))*(4.0*CL1(M)-1.0)-(1.0-S(K)**2))
DL1(11,N)=2.0*CL2(M)*(1.0-S(K))
DL1(12,N)=0.0
DL1(13,N)=0.0
DL1(14,N)=0.0
DL1(15,N)=2.0*CL3(M)*(1.0-S(K))
DL2(1,N)=0.0
DL2(2,N)=2.0*CL1(M)*(1.0+S(K))

```



```

DL2(3,N)= 0.50*((1.0+S(K))*(4.0*CL2(M)-1.0)-(1.0-S(K)**2))
DL2(4,N)= 2.0*CL3(M)*(1.0+S(K))
DL2(5,N)=0.0
DL2(6,N)=0.0
DL2(7,N)=0.0
DL2(8,N)= (1.0-S(K)**2)
DL2(9,N)=0.0
DL2(10,N)=0.0
DL2(11,N)= 2.0*CL1(M)*(1.0-S(K))
DL2(12,N)=0.50*((1.0-S(K))*(4.0*CL2(M)-1.0)-(1.0-S(K)**2))
DL2(13,N)= 2.0*CL3(M)*(1.0-S(K))
DL2(14,N)=0.0
DL2(15,N)=0.0
DL3(1,N)=0.0
DL3(2,N)=0.0
DL3(3,N)=0.0
DL3(4,N)= 2.0*CL2(M)*(1.0+S(K))
DL3(5,N)= 0.50*((1.0+S(K))*(4.0*CL3(M)-1.0)-(1.0-S(K)**2))
DL3(6,N)= 2.0*CL1(M)*(1.0+S(K))
DL3(7,N)=0.0
DL3(8,N)=0.0
DL3(9,N)= (1.0-S(K)**2)
DL3(10,N)=0.0
DL3(11,N)=0.0
DL3(12,N)=0.0
DL3(13,N)= 2.0*CL2(M)*(1.0-S(K))
DL3(14,N)=0.50*((1.0-S(K))*(4.0*CL3(M)-1.0)-(1.0-S(K)**2))
DL3(15,N)= 2.0*CL1(M)*(1.0-S(K))
DS(1,N)=0.50*(2.0*CL1(M)**2-CL1(M))+CL1(M)*S(K)
DS(2,N)= 2.0*CL1(M)*CL2(M)
DS(3,N)=0.50*(2.0*CL2(M)**2-CL2(M))+CL2(M)*S(K)
DS(4,N)= 2.0*(2.0*CL2(M)*CL3(M)
DS(5,N)=0.50*(2.0*CL3(M)**2-CL3(M))+CL3(M)*S(K)
DS(6,N)= 2.0*CL1(M)*CL3(M)
DS(7,N)= -2.0*CL1(M)*S(K)
DS(8,N)= -2.0*CL2(M)*S(K)
DS(9,N)= -2.0*CL3(M)*S(K)
DS(10,N)= -50*(2.0*CL1(M)**2-CL1(M))+CL1(M)*S(K)
DS(11,N)= -2.0*CL1(M)*CL2(M)
DS(12,N)= -50*(2.0*CL2(M)**2-CL2(M))+CL2(M)*S(K)
DS(13,N)= -2.0*CL2(M)*CL3(M)
DS(14,N)= -50*(2.0*CL3(M)**2-CL3(M))+CL3(M)*S(K)
DS(15,N)= -2.0*CL1(M)*CL3(M)

```

```

20 CONTINUE
10

```

```

DO 510 L=1,NUMEL
DO 475 J=1,NELDOF
DO 470 J=1,NELDOF

```



```

470 G(I,J)=0.0
475 CONTINUE
DO 480 JJ=1,NELDOF
NN=CONN(L,JJ)
XX(JJ)=X(NN)
YY(JJ)=P(NN)
ZZ(JJ)=Z(NN)
480 CONTINUE
A5=2.0/(ZZ(1)-ZZ(15))
DETJJ=0.0
DO 200 K=1,3
DETJ1=0.0
DO 210 M=1,7
IF(K-2)31,41,51
31 N=M
GO TO 61
41 N=M+7
GO TO 61
51 N=M+14
61 CONTINUE
T1=DL1(1,N)*XX(1)+DL1(2,N)*XX(2)-DL3(4,N)*XX(4)-DL3(5,N)*XX(5)
T2=DL1(7,N)*XX(7)-DL3(9,N)*XX(9)+DL1(13,N)*XX(10)+DL1(11,N)*XX(11)
T3=(DL1(6,N)-DL3(6,N))*XX(6)-DL3(13,N)*XX(13)-DL3(14,N)*XX(14)
T4=(DL1(15,N)-DL3(15,N))*XX(15)
DJ1=T1+T2+T3+T4
D1=DL1(1,N)*YY(1)+DL1(2,N)*YY(2)-DL3(4,N)*YY(4)-DL3(5,N)*YY(5)
D2=DL1(7,N)*YY(7)-DL3(9,N)*YY(9)+DL1(10,N)*YY(10)+DL1(11,N)*YY(11)
D3=(DL1(6,N)-DL3(6,N))*YY(6)-DL3(13,N)*YY(13)-DL3(14,N)*YY(14)
D4=(DL1(15,N)-DL3(15,N))*YY(15)
DJ2=D1+D2+D3+D4
T5=DL2(2,N)*XX(2)+DL2(3,N)*XX(3)-DL3(5,N)*XX(5)-DL3(6,N)*XX(6)
T6=DL2(8,N)*XX(8)-DL3(9,N)*XX(9)+DL2(4,N)*XX(4)
T7=DL2(11,N)*XX(11)+DL2(12,N)*XX(12)+(DL2(13,N)-DL3(13,N))*XX(13)
T8=-DL3(14,N)*XX(14)-DL3(15,N)*XX(15)
DJ21=T5+T6+T7+T8
D5=DL2(2,N)*YY(2)+DL2(3,N)*YY(3)-DL3(5,N)*YY(5)-DL3(6,N)*YY(6)
D6=DL2(8,N)*YY(8)-DL3(9,N)*YY(9)+DL2(4,N)*YY(4)
D7=DL2(11,N)*YY(11)+DL2(12,N)*YY(12)+(DL2(13,N)-DL3(13,N))*YY(13)
D8=-DL3(14,N)*YY(14)-DL3(15,N)*YY(15)
DJ22=D5+D6+D7+D8
DETJ(N)=DJ1+DJ22-DJ12+DJ21
DETJ1=DETJ(N)*WL(M)+DETJ1
CONTINUE
210 DETJJ=DETJJ+WS(K)*DETJ1
200 CONTINUE
DO 600 J=1,15
DO 610 I=1,15

```



```

FG=0.0
DO 660 K=1,3
F=0.0
DO 670 M=1,7
IF(K-2)620,630,640
620 N=M
GO TO 650
630 N=M+7
GO TO 650
640 N=M+14
650 CONTINUE
F=F+WL(M)*FN(J,N)*FN(I,N)*DETJ(N)
670 CONTINUE
FG=FG+WS(K)*F
660 CONTINUE
G(J,I)=FG/A5
610 CONTINUE
600 ** INSERT ELEMENT MATRIX INTO SYSTEM MATRIX *****
DO 505 K=1,NELDOF
KK=CONN(L,K)
IF(KK.GT.NNZ) GO TO 505
KKK=JA(KK)
LLL=JB(KK)-1
DO 500 I=1,NELDOF
II=CONN(L,I)
IF(II.GT.NNZ) GO TO 500
DO 490 M=1,KK<
MM=LLL+M
KKM=NAME(MM)
IF(II.EQ.KKM) GO TO 495
490 CONTINUE
495 BIGG(MM)=BIGG(MM)+G(K,I)
500 CONTINUE
505 CONTINUE
510 CONTINUE
RETURN
END

```

 THIS IS THE MAIN PROGRAM FOR THE SOLUTION OF THE FIELD EQUATIONS.
 THE DATA FROM THE TAPE IS READ OUT, NODAL CONSTANTS ARE EVALUATED,
 THE INTEGRATION PACKAGE IS INITIATED, ETC.

```

REAL*8 SIGF, SIGA, C1, C2, C4, C5
REAL*8 GGG, BIGG, BIGGG, BIGH
REAL*8 WL, WS, S, CL1, CL2, CL3, FN, DL1, DL2, DL3, DS, DETJ
INTEGER*2 NAME, JA, JB, NNZ, JC, NUMNP, NUMEL, NELDOF, CONN
COMMON/GTRY2/NAME(4615), JA(183), JB(184), NNZ, JC, NUMNP, NUMEL, NELDOF,
1CONN(128,15)
COMMON/GTRY1/X(505), P(505), Z(505)
COMMON/XOCAL/KX(15), YY(15), ZZ(15), PPSI(15)
DIMENSION W(700), Y(7,183)
COMMON/DELAY/SUM(183), PSI(93)
COMMON/TEMP/C3(183), C6(15)
DIMENSION SIGF(93), SIGA(183)
COMMON/COEF/C1(183), C2(183), C4(183), C5(183)
COMMON/GGGMAT/GGG(15,15)
COMMON/SYSMT1/BIGG(4615)
COMMON/SYSMT2/BIGGG(4615)
COMMON/SYSMT3/BIGH(4615)
COMMON/GAUSS/WL(7), WS(5), S(5), CL1(7), CL2(7), CL3(7), FN(15,21), DL1(1
15,21), DL2(15,21), DL3(15,21), DS(15,21), DETJ(21)
NNZ=183
NUMEL=128
NUMNP=505
NELDOF=15
JC=4615
FMAX=1.0E10
DO 771 I=1, NUMEL
  READ(4) (CONN(I, J), J=1, NELDOF)
771 CONTINUE
  (JA(I), I=1, NNZ)
  READ(4) (JB(I), I=1, NNZ)
  READ(4) (NAME(I), I=1, JC)
  READ(4) (X(I), I=1, NUMNP)
  READ(4) (P(I), I=1, NUMNP)
  READ(4) (Z(I), I=1, NUMNP)
  READ(4) (BIGG(I), I=1, JC)
  READ(4) (BIGGG(I), I=1, JC)
DO 772 I=1, 3
  READ(4) WS(I), S(I)
772 CONTINUE

```

M-010
M-015
M-020
M-025
M-055
M-060

M-075
M-080
M-085
M-095
M-100

M-105


```

DO 773 I=1,7
READ(4) WL(I),CL1(I),CL2(I),CL3(I)
773 CONTINUE
DO 781 I=1,NELDOF
DO 782 J=1,21
READ(4) FN(I,J),DL1(I,J),DL2(I,J),DL3(I,J),DS(I,J)
782 CONTINUE
781 CONTINUE
DO 460 I=1,NNZ
Y(1,I)=FMAX
Y(2,I)=0.0
SUM(I)=0.0
IF(I.GT.93) GJ TO 460
PSI(I)=FMAX
460 CONTINUE
WRITE(6,3)
3 FORMAT(10F10.4) SIGF(I)=0.0057450, SIGA(I)=0.014010,0.0080 .)
V=4.800E07
Q=0.4349710
BETA=0.006420
B=-0.00400
DO 445 I=1,NNZ GJ TO 420
IF(I.GT.93) GJ TO 420
D=0.9130
SIGF(I)=0.0057450
SIGA(I)=0.014010
ZNU=2.54
C1(I)=V*D
C2(I)=V*SIGA(I)-V*(1.0-BETA)*SIGF(I)*ZNU
C3(I)=(2.0179325E-13)*SIGF(I)
C4(I)=-V*(1.0-BETA)*SIGA(I)*B
C5(I)=-V*BETA*Q*ZNU*SIGF(I)
GO TO 445
420 D=1.200
SIGA(I)=0.00800
C1(I)=V*D
C2(I)=V*SIGA(I)
C3(I)=0.0
C4(I)=0.0
C5(I)=0.0
445 CONTINUE
TEND=0.0010
NY=NNZ
NL=0
H=1.0E-19
T=0.0
HMIN=1.0E-20
HMAX=0.10

```

M-520
M-535
M-545
M-540
M-550

M-565
M-570
M-575
M-580
M-585
M-590


```

SDESOL. JSKF > 0 INDICATES A CONTINUATION OF THE SDE
PREVIOUS CALL TO SDESOL. JSKF < -1 MAY HAVE RESULTS SDE
FROM THE USER NEGLECTING TO TEST FOR ERROR RETURNS SDE
FROM SDESOL. BECAUSE OF THIS POSSIBILITY, JSKF < -1 SDE
RESULTS IN TERMINATION OF THE RUN WITH THE SDE
APPROPRIATE COMMENT. SDE
ON OUTPUT, JSKF CONSISTS OF TWO DIGITS AND SIGN, SDE
+ OR - QP. Q IS THE ORDER OF THE FORMULA CURRENTLY SDE
BEING USED. P INDICATES THE TYPE OF RETURN, AS SDE
FOLLOWS. SDE
JSKF > 0, P = 1 IS THE NORMAL RETURN SDE
JSKF < 0, P IS AN ERROR RETURN, WITH THE FOLLOWING SDE
MEANINGS. SDE
P = 1 ERROR TEST FAILURE FOR H > HMIN SDE
P = 3 CORRECTOR FAILED TO CONVERGE FOR H > HMIN SDE
P = 4 ORDER METHOD FAILED TO CONVERGE FOR FIRST SDE
P = 5 ORDER METHOD FROM SUBROUTINE NUTSL SDE
P = 6 ERROR RETURN FROM SUBROUTINE DERVAL SDE
MAXDER - MAXIMUM ORDER DERIVATIVE THAT SHOULD BE USED IN SDE
METHOD. IT MUST BE NO GREATER THAN SIX. SDE
IPRT - INTERNAL PRINT CONTROL INDICATOR FOR LDASUB. SDE
IPRT = 0 NO PRINT SDE
IPRT > 0 PRINT COUNTERS, STEPSIZE, CURRENT TIMES SDE
AND VALUES OF DEPENDENT VARIABLES AT SDE
EACH STEP. SDE
H - CURRENT STEPSIZE. AN INITIAL VALUE MUST BE SUPPLIED SDE
3JT NEED NOT BE THE ONE WHICH MUST BE USED, SINCE THE SDE
SUBROUTINE WILL CHOOSE A SMALLER ONE IF NECESSARY TO SDE
KEEP THE ERROR PER STEP SMALLER THAN THE SPECIFIED SDE
VALUE. IT IS BETTER TO UNDERESTIMATE THE INITIAL SDE
STEPSIZE THAN TO OVERESTIMATE IT. THE STEPSIZE IS SDE
VARIABLELY NOT CHANGED BY THE USER. SDE
HMIN - MINIMUM STEPSIZE ALLOWED SDE
HMAX - MAXIMUM STEPSIZE ALLOWED SDE
RMSEPS - THE ERROR TEST CONSTANT. THE ROOT-MEAN-SQUARE OF SDE
THE SINGLE STEP ERROR ESTIMATES, ER(I), DIVIDED BY SDE
YMAX(I) = (MAXIMUM TO CURRENT TIME OF Y(I)) MUST BE SDE
LESS THAN EPS. THE STEPSIZE AND/OR THE ORDER SDE
ARE VARIED TO ACHIEVE THIS. SDE
W - SCATCH STORAGE ARRAY. MUST BE AT LEAST 13*NY + 5*NL SDE
LOCATIONS, PLUS THOSE REQUIRED FOR STORAGE OF THE SDE
MATRIX PW. (SEE DESCRIPTION OF SUBROUTINE JACMAT). SDE
THE STORAGE OF PW WILL NORMALLY REQUIRE NO MORE THAN SDE
N*2 + 2*N LOCATIONS, AND IF COMPACT STORAGE TECH- SDE
NIQUES ARE USED, CAN BE MUCH FEWER. SDE

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1HMIN,HMAX,RMSEPS,SAVE,YLSV,YMAX,ER,ESV,FI,DY,PW)
SUBROUTINE LDASUB IS A MODIFICATION OF SUBROUTINE DFASUB
WHICH IS DUE TO R. L. BROWN AND C. W. GEAR. DFASUB IS DOCUMENTED
IN THE REPORT
DOCUMENTATION FOR DFASUB--
BY R. L. BROWN AND C. W. GEAR
REPORT UIUCDCS-R-73-575, JULY 1973
UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
URBANA, ILLINOIS 61801
THIS REPORT IS AVAILABLE FROM THE NATIONAL TECHNICAL INFORMATION
SERVICE OF THE U. S. DEPARTMENT OF COMMERCE UNDER ACCESSION NUMBER
C00-1459-225.
THE MODIFICATION HERE IS DOCUMENTED IN THE REPORT
A PROGRAM FOR THE NUMERICAL SOLUTION OF LARGE SPARSE SYSTEMS OF
ALGEBRAIC AND IMPLICITLY DEFINED STIFF DIFFERENTIAL EQUATIONS
BY RICHARD FRANK
REPORT NPS53FE76051, MAY 1976
NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA 93940
-----
THE CALLING SEQUENCE FOR LDASUB IS
CALL LDASUB(Y,YL,T,TEND,V,NY,M,JSTART,KFLAG,MAXOR,IPRT,H,HMIN,
HMAX,RMSEPS,SAVE,YLSV,YMAX,ER,ESV,FI,DY,PW)
WHERE THE PARAMETERS ARE DEFINED AS FOLLOWS.
Y - ARRAY DIMENSIONED (7,NY). THIS ARRAY CONTAINS THE
DEPENDENT VARIABLES AND THEIR SCALED DERIVATIVES.
Y(J+1,I) CONTAINS THE J-TH DERIVATIVE OF THE I-TH VARIABLE
TABLE TIMES H**J/J-FACTORIAL, WHERE H IS THE CURRENT
STEP SIZE. ON FIRST ENTRY THE CALLER SUPPLIES THE
INITIAL VALUES OF EACH VARIABLE IN Y(1,I) AND AN
ESTIMATE OF THE INITIAL VALUES OF THE DERIVATIVES
IN Y(2,I). ON SUBSEQUENT ENTRIES IT IS ASSUMED THAT
THE ARRAY HAS NOT BEEN CHANGED. TO INTERPOLATE TO
NON-MESH POINTS, THESE VALUES CAN BE USED AS FOLLOWS.
IF H IS THE CURRENT STEP SIZE AND VALUES AT TIME T+E
NEEDED, LET S = E/H AND THEN
NQ SUM Y(J+1,I)*S**J
I-TH VARIABLE AT T+E IS J=0
THE VALUE OF NQ IS OBTAINED IN THE CALLING PROGRAM

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3Y  NQ = JSTART.
-   ARRAY OF NL = N - NY VARIABLES WHICH APPEAR LINEARLY.
-   THE USER SUPPLIES INITIAL VALUES FOR THESE VARIABLES.
-   CURRENT VALUE OF THE INDEPENDENT VARIABLE (TIME)
-   END TIME
-   TOTAL NUMBER OF VARIABLES
-   NUMBER OF DIFFERENTIAL EQUATIONS AND NONLINEAR
-   VARIABLES.
-   NUMBER OF VARIABLES INCLUDED IN THE ERROR TEST.
-   THIS NUMBER CAN BE NO GREATER THAN NY. IF IT IS
-   GREATER THAN NY, NY VARIABLES ARE USED IN THE ERROR
-   TEST.
-   INPUT AND OUTPUT INDICATOR.
-   ON INPUT
-   ON INPUT <0
-   THIS INDICATES A RE-START FROM A PREVIOUS
-   POINT FOLLOWING A TERMINATION OF THE RUN OR
-   SOLUTION OF ANOTHER PROBLEM DURING THE SAME
-   RUN. PARAMETERS IN THE CALLING SEQUENCE
-   MUST HAVE BEEN PRESERVED FROM THE PREVIOUS
-   USE, PARTICULARLY THE ARRAYS
-   SAVE, YLSV, ESV, AND PW.
-   THESE ARRAYS MUST BE SAVED AFTER A CALL
-   TO SUBROUTINE LDASAV, WHICH ALSO SAVES
-   NECESSARY PARAMETERS, INTERNAL TO LDASUB.
-   INDICATES AN INITIAL CALL TO LDASUB. THE
-   ROUTINE INITIALIZES ITSELF, SCALES THE
-   DERIVATIVES IN Y(2,1) AND THEN PERFORMS THE
-   INTEGRATION UNTIL T > TEND.
-   INDICATES THE SOLUTION IS TO BE CONTINUED.
-   AFTER THE INITIAL ENTRY IT IS NEITHER
-   DESIRABLE NOR NECESSARY TO RE-ENTER WITH
-   JSTART = 0, SINCE THIS RE-INITIALIZES
-   THE CODE, BEGINNING WITH A FIRST ORDER
-   METHOD AGAIN.
-   ON OUTPUT, JSTART IS SET TO THE VALUE OF NQ, THE
-   ORDER OF THE FORMULA CURRENTLY BEING USED.
-   THE COMPLETION CODE INDICATOR, WITH THE FOLLOWING
-   MEANINGS
-   +1 THE INTEGRATION WAS SUCCESSFUL
-   -1 ERROR TEST FAILED FOR H > HMIN
-   -3 CORRECTOR FAILED TO CONVERGE FOR H > HMIN
-   -4 CORRECTOR FAILED TO CONVERGE FOR FIRST
-   ORDER METHOD
-   -5 ERROR RETURN FROM SUBROUTINE NUTSL
-   MAXOR ORDER DERIVATIVE THAT SHOULD BE USED IN THE
-   METHOD. IT MUST BE NO GREATER THAN SIX. IF IT IS
-   GREATER THAN SIX, THE MAXIMUM ORDER USED WILL BE SIX.

```

```

LDA 500
LDA 510
LDA 520
LDA 530
LDA 540
LDA 550
LDA 560
LDA 570
LDA 580
LDA 590
LDA 600
LDA 610
LDA 620
LDA 630
LDA 640
LDA 650
LDA 660
LDA 670
LDA 680
LDA 690
LDA 700
LDA 710
LDA 720
LDA 730
LDA 740
LDA 750
LDA 760
LDA 770
LDA 780
LDA 790
LDA 800
LDA 810
LDA 820
LDA 830
LDA 840
LDA 850
LDA 860
LDA 870
LDA 880
LDA 890
LDA 900
LDA 910
LDA 920
LDA 930
LDA 940
LDA 950
LDA 960
LDA 970

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CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

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C      DATA PERT/4.,9.,16.,25.,36.,49.,9.,16.,25.,36.,49.,64.,1.,1.,.25,
C      12.7889E-2,1.70569E-3,6.83929E-5/
C      -----
C      THE ENTRIES IN THE COF ARRAY ARE THE CJEFFICIENTS FOR THE STIFFLY
C      STABLE METHODS USED IN THIS PROGRAM AND ARE TO BE THE MACHINE
C      PRECISION EQUIVALENTS OF THE FOLLOWING CONSTANTS.
C
C      -1/2, -1/2
C      -1/6, -1, -1/6
C      -25/12, -35/24, -5/12, -1/24
C      -137/60, -15/8, -17/24, -1/8, -1/120
C      -147/60, -203/90, -49/48, -35/144, -7/240, -1/720
C      -----
C      DATA CJF/-1.,-1.5,-.5,-1.833333,-1.,-.1666667,-2.083333,-1.458333,
C      1-.4166667,-.0416667,-2.283333,-1.875,-.7083333,-.125,-.008333333,
C      2-2.45,-2.255556,-1.020833,-.2430556,-.32916667,-.001388889/
C      IF (JSTART) 100,110,150
C      -----
C      IF THIS IS A RESTART ENTRY, RESTORE Y AND YL FROM THE SAVE AND
C      YLSV ARRAYS, WHERE THEY WERE SAVED BY A PREVIOUS CALL TO LDASAV.
C      -----
C      100 CALL COPYZ (Y,SAVE,LCOPYY)
C      CALL COPYZ (YL,YLSV,LCOPYL)
C      GO TO 150
C      -----
C      IF THIS IS THE FIRST CALL, INITIALIZE YMAX, SCALE DERIVATIVES, AND
C      INITIALIZE INDICATORS AND SET ORDER TO ONE.
C      FOR DOUBLE PRECISION, SET LCOPYL = 14*NY AND LCOPYL = 2*NL IF
C      SUBROUTINE COPYZ IS IN SINGLE PRECISION.
C      -----
C      110 NL = N-NY
C      LCOPYY = 7*NY
C      LCOPYL = NL
C      M1 = MINO(M,NY)
C      EPS = SQRT(FLJAT(M1))*RMSEPS
C      MAXDER = MINO(MAXOR,6)
C      IF (IPRT.LE.0) GO TO 120
C      PRINT 3, N,NL,RMSEPS,TEND,H
C      PRINT 4
C      120 NS = 0
C      NW = 0
C
C      DO 130 J=1,NY

```



```

C      YMAX(J) = AMAX1(1.,ABS(Y(1,J)))
130  Y(2,J) = Y(2,J)*H
C
C      NQ = 1
C      BR = 1.
C      ASSIGN 190 TO IRET
C-----
C      SET COEFFICIENTS FOR THE ORDER CURRENTLY BEING USED.
C      IS A TEST FOR ERRORS OF THE CURRENT ORDER NQ
C      EUP IS TO TEST FOR INCREASING THE ORDER, EDWN FOR DECREASING THE
C      ORDER.
C-----
140  K = NQ*(NQ-1)/2
C      CALL COPYZ (A(2),COF(K+1),NQ)
C      K = NQ+1
C      IDQUB = NQ
C      ENQ1 = .5/NQ
C      ENQ2 = .5/K
C      ENQ3 = .5/(NQ+2)
C      PEP SH = EPS**2
C      E = PERT(NQ,1)*PEP SH
C      EUP = PERT(NQ,2)*PEP SH
C      EDWN = PERT(NQ,3)*PEP SH
C      BND = (EPS*ENQ3)**2
C      IWEVAL = 1
C      GO TO IRET, (190,200,490,570)
150  IF (H.EQ.HNEW) GO TO 190
C-----
C      IF CALLER HAS CHANGED H, RESCALE DERIVATIVES TO REFLECT THAT HNEW
C      WAS USED ON THE LAST CALL.
C-----
C      R = H/HNEW
C      ASSIGN 190 TO IRET
C      GO TO 610
C-----
C      SET JSTART TO NQ, THE CURRENT ORDER OF THE METHOD, BEFORE EXIT,
C      AND SAVE THE CURRENT STEP SIZE IN HNEW.
C-----
160  JSTART = NQ
C      HNEW = H
C      RETURN
170  NS = NS+1
C      IF (IPRT.LE.0) GO TO 180
C-----
C      PRINT DATA IF DESIRED BY USER
C-----
C      PRINT 1, NS, NQ, H, T, (Y(1,I), I=1, NY)
C      IF (NL.GT.0) PRINT 2, (YL(I), I=1, NL)

```

```

LDA 1940
LDA 1950
LDA 1960
LDA 1970
LDA 1980
LDA 1990
LDA 2000
LDA 2010
LDA 2020
LDA 2030
LDA 2040
LDA 2050
LDA 2060
LDA 2070
LDA 2080
LDA 2090
LDA 2100
LDA 2110
LDA 2120
LDA 2130
LDA 2140
LDA 2150
LDA 2160
LDA 2170
LDA 2180
LDA 2190
LDA 2200
LDA 2210
LDA 2220
LDA 2230
LDA 2240
LDA 2250
LDA 2260
LDA 2270
LDA 2280
LDA 2290
LDA 2300
LDA 2310
LDA 2320
LDA 2330
LDA 2340
LDA 2350
LDA 2360
LDA 2370
LDA 2380
LDA 2390
LDA 2400
LDA 2410

```



```

180 CONTINUE
IF (KFLAG.LT.0) GO TO 160
IF (T.GE.TEND) GO TO 160
C-----
C TAKE ANOTHER STEP IF T < TEND
C-----
C JSTART = 1
C-----
C SAVE DATA FOR TRIAL WITH A SMALLER TIMESTEP IF THIS STEP FAILS
C-----
190 CALL COPYZ (SAVE,Y,LCOPYZ)
CALL COPYZ (YLSV,YL,LCOPYL)
RACUM = 1.
KFLAG = 1
HOLD = H
NQOLD = NQ
TOLD = T
T = T+H
HINV = 1./H
C-----
C COMPUTE PREDICTED VALUES BY EFFECTIVELY MULTIPLYING DERIVATIVE
C VECTOR BY PASCAL TRIANGLE MATRIX
C-----
C DO 210 J=2,K
C J3 = K+J-1
C
C DO 210 J1=J,K
C J2 = J3-J1
C
C DO 210 I=1,NY
210 Y(J2,I) = Y(J2,I)+Y(J2+1,I)
C
C DO 220 I=1,NY
220 ER(I) = 0.
C-----
C DO UP TO THREE CORRECTOR ITERATIONS. CONVERGENCE IS OBTAINED WHEN
C CHANGES ARE LESS THAN BND WHICH IS DEPENDENT ON THE ERROR TEST
C CONSTANT. THE SUM OF CORRECTIONS IS ACCUMULATED IN ER(I). IT IS
C EQUAL TO THE K-TH DERIVATIVE OF Y TIMES H**K/(K-FACTORIAL*A(K)),
C AND THUS IS PROPORTIONAL TO THE ACTUAL ERRORS TO THE LOWEST POWER
C OF H PRESENT, WHICH IS H**K.
C-----
C DO 270 L=1,3
CALL DIFFUN (Y,YL,T,HINV,DY)
C-----

```



```

C C C C C C C
IF (IWEVAL.LT.1) GO TO 230
IF THERE HAS BEEN A CHANGE OF ORDER OR THERE HAS BEEN TROUBLE
WITH CONVERGENCE, PW IS RE-EVALUATED PRIOR TO STARTING THE
CORRECTOR ITERATION. IWEVAL IS THEN SET TO -1 AS AN INDICATOR
THAT IT HAS BEEN DONE. NEWPW IS SET NONZERO TO INDICATE TO
SUBROUTINE NUTSL THAT A NEW PW HAS BEEN PROVIDED.
LDA 2900
LDA 2910
LDA 2920
LDA 2930
LDA 2940
LDA 2950
LDA 2960
LDA 2970
LDA 3250
LDA 2980
LDA 2990
LDA 3000
LDA 3010
LDA 3020
LDA 3030
LDA 3040
LDA 3050
LDA 3060
LDA 3070
LDA 3080
LDA 3090
LDA 3100
LDA 3110
LDA 3120
LDA 3130
LDA 3140
LDA 3150
LDA 3160
LDA 3170
LDA 3180
LDA 3190
LDA 3200
LDA 3210
LDA 3220
LDA 3230
LDA 3240
LDA 3260
LDA 3270
LDA 3280
LDA 3290
LDA 3300
LDA 3310
LDA 3320
LDA 3330
LDA 3340
LDA 3350
LDA 3360
LDA 3370

-----
CALL JACMAT (Y, YL, T, HINV, A(2), N, NY, EPS, DY, F1, PW)
KFLAG = 1
IWEVAL = -1
NW = NW+1
NEWPW = 1
230 CALL NUTSL (PW, DY, F1, N, NY, EPS, YMAX, NEWPW, KRRET)
IF (KRRET.NE.0) GO TO 600
IF (NL.LE.0) GO TO 250
C
DO 240 I=1,NL
240 YL(I) = YL(I)-F1(I+NY)
C
250 CONTINUE
DEL = 0.
C
DO 260 I=1,NY
Y(1,I) = Y(1,I)-F1(I)
Y(2,I) = Y(2,I)+A(2)*F1(I)
ER(I) = ER(I)+F1(I)
DEL = DEL+(F1(I)/AMAX1(YMAX(I),ABS(Y(1,I))))**2
260 CONTINUE
C
IF (L.GE.2) BR = AMAX1(.9*BR,DEL/DEL1)
DEL1 = DEL
IF (AMIN1(DEL,BR*DEL*2.).LE.BND) GO TO 330
270 CONTINUE
C
THE CORRECTOR ITERATION FAILED TO CONVERGE IN 3 TRIES. VARIOUS
POSSIBILITIES ARE CHECKED FOR. IF H IS ALREADY HMIN AND PW HAS
ALREADY BEEN RE-EVALUATED, A NO CONVERGENCE EXIT IS TAKEN.
OTHERWISE THE MATRIX PW IS RE-EVALUATED AND/OR (IN THAT ORDER) THE
STEP IS REDUCED TO TRY AND GET CONVERGENCE.
T = TOLD
IF (IWEVAL) 280,300,290
280 IF (H.LE.HMIN*1.00001) GO TO 310
290 RACUM = RACUM*.25
300 CONTINUE
GO TO 560

```



```

C      DO 390 I=1,NY
C      390 SAVE(2,I) = Y(2,I)
C      LDA 3850
C      400 KFLAG = KFLAG-2
C      IF (H.LE.HMIN) GO TO 550
C      T = TOLD
C      IF (KFLAG.LE.-5) GO TO 530
C      410 PR2 = (D/E)**NQ2*1.2
C      L = 0
C      IF (NQ.LE.1) GO TO 430
C      D = 0.
C      DO 420 J=1,M1
C      YM = AMAX1(ABS(Y(1,J)),YMAX(J))
C      420 D = D+((Y(K,J)/YM)**2)
C      LDA 4000
C      PR1 = (D/EDWN)**ENQ1*1.3
C      IF (PR1.GE.PR2) GO TO 430
C      PR2 = PR1
C      L = -1
C      430 IF (KFLAG.LT.0.JR.NQ.GE.MAXDER) GO TO 450
C      D = 0
C      DO 440 J=1,M1
C      YM = AMAX1(ABS(Y(1,J)),YMAX(J))
C      440 D = D+((ER(J)-ESV(J))/YM)**2
C      LDA 4010
C      PR1 = (D/EUP)**ENQ3*1.4
C      IF (PR1.GE.PR2) GO TO 450
C      PR2 = PR1
C      L = 1
C      450 R = 1./AMAX1(PR2,1.E-5)
C      IF (KFLAG.LT.0.JR.R.GE.1.1) GO TO 460
C      IDOUB = 9
C      GO TO 510
C      460 NEWQ = NQ+L
C      K = NEWQ+1
C      IF (NEWQ.LE.NQ) GO TO 480
C      R1 = A(NEWQ)/FLOAT(NEWQ)
C      DO 470 J=1,NY
C      470 Y(K,J) = ER(J)*R1
C      480 CONTINUE
C      IF THE STEP WAS OKAY, SCALE THE Y VARIABLES IN ACCORDANCE
C      WITH THE NEW VALUE OF H. IF KFLAG < 0, HOWEVER, USE THE
C      SAVED VALUES (IN SAVE AND YLSV). IN EITHER CASE, IF THE ORDER

```



```

C      HAS CHANGED IT IS NECESSARY TO FIX CERTAIN PARAMETERS BY CALLING
C      THE PROGRAM SEGMENT AT STATEMENT NUMBER 140.
C      -----
C      IDOUB = NQ
C      IF (NEWQ.EQ.NQ) GO TO 490
C      NQ = NEWQ
C      ASSIGN 490 TO IRET
C      GO TO 140
C      IF (KFLAG.GT.J) GO TO 500
C      RACUM = RACUM*R
C      GO TO 560
C      R = AMAX1(AMIN1(HMAX/H,R),HMIN/H)
C      H = H*R
C      IWEVAL = 1
C      ASSIGN 510 TO IRET
C      GO TO 610
C      510 DO 520 I=1,M1
C      520 YMAX(I) = AMAX1(ABS(Y(1,I)),YMAX(I))
C      GO TO 170
C      -----
C      THE ERROR TEST HAS NOW FAILED THREE TIMES, SO THE DERIVATIVES ARE
C      IN BAD SHAPE. RETURN TO FIRST ORDER METHOD AND TRY AGAIN. OF
C      COURSE, IF NQ = 1 ALREADY, THEN THERE IS NO HOPE AND WE EXIT WITH
C      KFLAG = -4.
C      -----
C      530 IF (NQ.EQ.1) GO TO 540
C      NQ = 1
C      IDOUB = 1
C      ASSIGN 570 TO IRET
C      GO TO 140
C      540 NQOLD = 1
C      KFLAG = -4
C      GO TO 320
C      550 KFLAG = -1
C      GO TO 170
C      -----
C      THIS SECTION RESTORES THE SAVED VALUES OF Y AND YL, SCALING THE
C      Y DERIVATIVES AS NECESSARY, AND THEN RETURNS TO THE PREDICTOR LOOP
C      -----
C      560 H = HOLD*RACUM
C      H = AMAX1(HMIN,AMIN1(H,HMAX))
C      570 RACUM = H/HOLD
C      R1 = 1.
C      DO 580 J=2,K
C      R1 = R1*RACUM
C      580
C      590
C      600
C      610
C      620
C      630
C      640
C      650
C      660
C      670
C      680
C      690
C      700
C      710
C      720
C      730
C      740
C      750
C      760
C      770
C      780
C      790
C      800
C      810
C      820
C      830
C      840
C      850
C      860
C      870
C      880
C      890
C      900
C      910
C      920
C      930
C      940
C      950
C      960
C      970
C      980
C      990
C      1000

```



```

C          DO 580 I=1,NY
580      Y(J,I) = SAVE(J,I)*R1
C
C          DO 590 I=1,NY
590      Y(1,I) = SAVE(1,I)
C          CALL COPYZ (YL,YLSV,LCOPYL)
IWEVAL=1
GO TO 200
600      KFLAG = 5
GO TO 160
C          -----
C          THIS SECTION SCALES THE Y DERIVATIVES BY R**J
C          -----
610      R1 = 1.
C
C          DO 620 J=2,K
R1 = R1*R
C
C          DO 620 I=1,NY
620      Y(J,I) = Y(J,I)*R1
C          GO TO IRET, (190,510)
C          -----
C          THIS SECTION ALLOWS FOR RESTARTS AFTER SOLVING ANOTHER PROBLEM, OR
HAVING TERMINATED THE CURRENT COMPUTER RUN. SUBROUTINE LDASAV
SAVES THE NECESSARY VALUES WHICH ARE INTERNAL TO LDASUB. FOR
DOUBLE PRECISION, WITH COPYZ IN SINGLE PRECISION, THE NUMBER OF
LOCATIONS TO BE SAVED AND RESTORED, LCPYS AND LCPYR, MUST BE
SET TO 58.
C          IT IS ASSUMED THAT IN ADDITION TO THE VARIABLES IN THE ARRAY A
IT IS ASSUMED THAT IN ADDITION TO THE VARIABLES IN THE ARRAY A
SAVED BY CALLING LDASAV, THE USER ALSO SAVES THE ARRAYS SAVE,
YLSV, YMAX, ESV, AND PW.
C          TO RESTART THE USER FIRST CALLS LDARST TO RESTORE THE VALUES SAVED
BY LDASAV, THEN RE-ENTERS LDASUB WITH JSTART < 0, AND WITH THE
OTHER PARAMETERS THE SAME AS RETURNED FROM THE LAST ENTRY TO
LDASUB, PARTICULARLY THOSE ARRAYS MENTIONED ABOVE.
C          -----
C          ENTRY LDASAV(SAV)
LCOPYS = 29
CALL COPYZ (SAV,A,LCOPYS)
CALL COPYZ (SAVE,Y,LCOPYY)
CALL COPYZ (YLSV,YL,LCOPYL)
RETURN
C

```

```

LDA 4820
LDA 4830
LDA 4840
LDA 4850
LDA 4860
LDA 4870
LDA 4880
LDA 4890
LDA 4900
LDA 4910
LDA 4920
LDA 4930
LDA 4940
LDA 4950
LDA 4960
LDA 4970
LDA 4980
LDA 4990
LDA 5000
LDA 5010
LDA 5020
LDA 5030
LDA 5040
LDA 5050
LDA 5060
LDA 5070
LDA 5080
LDA 5090
LDA 5100
LDA 5110
LDA 5120
LDA 5130
LDA 5140
LDA 5150
LDA 5160
LDA 5170
LDA 5180
LDA 5190
LDA 5200
LDA 5210
LDA 5220
LDA 5230
LDA 5240
LDA 5250
LDA 5260
LDA 5270
LDA 5280
LDA 5290

```



```

ENTRY LDARST(SAV)
LCOPYR = 29
CALL COPYZ (A, SAV, LCOPYR)
RETURN
-----
1 FORMAT (2I5, I2, 1P2E10.2, 7E14.6 / (32X, 7E14.6))
2 FORMAT (32X, 1P7E14.6)
3 FORMAT (11N = , I3, , NL = , I3, , RMSEPS = , 1PE9.2, , TEND = ,
1,E9.2, , H = , E9.2 //)
4 FORMAT (11NS NW Q H , 8X, , T , 8X, , Y(1,*) AND YL(*)' //)
END
-----
SUBROUTINE COPYZ(S, Y, L)
DIMENSION S(1), Y(1)
-----
THIS SUBROUTINE COPIES THE ARRAY Y, OF LENGTH L, INTO THE ARRAY S
-----
IF(L.LE.0) RETURN
DO 100 J=1, L
S(J) = Y(J)
100 RETURN
END
-----
SUBROUTINE Derval (Y, YL, T, N, NY, W, KERET)
THIS SUBROUTINE CALCULATES THE INITIAL VALUES OF THE DERIVATIVES
IN THE GENERAL CASE. IT IS WRITTEN SO THAT IT SHOULD WORK IF THE
FIRST NY EQUATIONS ALL INVOLVE DERIVATIVES. IT ATTEMPTS TO SOLVE
THE FIRST NY EQUATIONS USING NEWTON'S METHOD, BUT SINCE IT TRIES
TO EVALUATE DF/DY BY CALLING JACMAT IN SUCH A WAY AS TO MAKE THE
DF/DY TERM INSIGNIFICANT, IT IS POSSIBLE THAT IT MAY FAIL FOR THAT
REASON. IT MAY FAIL FOR OTHER REASONS, AS WELL. IF IT DOES FAIL
THE USER CAN SUPPLY HIS OWN VERSION OF Derval, OR MODIFY THIS
ROUTINE IN SUITABLE FASHION. THIS ROUTINE ASSUMES THAT VALUES OF
THE LINEAR VARIABLES HAVE BEEN SUPPLIED PREVIOUSLY. IF THOSE
MUST BE SOLVED FOR SIMULTANEOUSLY WITH THE DERIVATIVES, THE USER
MUST SUPPLY HIS OWN VERSION OF Derval.
-----
THE CALLING SEQUENCE FOR THIS SUBROUTINE IS
CALL Derval(Y, YL, T, N, NY, W, KERET)
WHERE THE PARAMETERS ARE DEFINED AS FOLLOWS

```

```

LDA 5300
LDA 5310
LDA 5320
LDA 5330
LDA 5340
LDA 5350
LDA 5360
LDA 5370
LDA 5380
LDA 5390
LDA 5400
LDA 5410
LDA 5420
LDA 5430
COP 10
COP 20
COP 30
COP 40
COP 50
COP 60
COP 70
COP 80
COP 90
COP 100
COP 110
COP 120
DER 10
DER 20
DER 30
DER 40
DER 50
DER 60
DER 70
DER 80
DER 90
DER 100
DER 110
DER 120
DER 130
DER 140
DER 150
DER 160
DER 170
DER 180
DER 190
DER 200

```



```

      IF (ER.LT.EPS2) GO TO 150
140  CONTINUE
      GO TO 170
      DO 160 I=1,NY
160  Y(2,I) = Y(3,I)
      RETURN
170  KERET = 1
      RETURN
      END
DER 690
DER 700
DER 710
DER 720
DER 730
DER 740
DER 750
DER 760
DER 770
DER 780
DER 790
DER 800

```

 THIS SUBROUTINE IS REQUIRED BY THE TIME INTEGRATION PACKAGE AND
 MUST BE SUPPLIED BY THE USER. ITS PURPOSE IS TO EVALUATE THE
 FUNCTION AT CURRENT VALUES OF THE VARIABLES.

```

SUBROUTINE DIFFUN(Y,YL,T,HINV,DY)
REAL*8 C1,C2,C3,C4,C5
REAL*8 BIGG,BIGG,BIGH
INTEGER*2 NAME,JA,JB,NNZ,JC,NUMNP,NUMEL,NELDOF,CONN
COMMON/GTRY2/NAME(4615),JA(183),JB(184),NNZ,JC,NUMNP,NUMEL,NELDOF,
1  CONN(128,15)
DIMENSION Y(7,1),YL(1),DY(1)
COMMON/DELAY/SUM(183),PSI(93)
COMMON/COEF/C1(183),C2(183),C4(183),C5(183)
COMMON/SYSMT1/BIGG(4615)
COMMON/SYSMT2/BIGG(4615)
COMMON/SYSMT3/BIGH(4615)
DATA TOLD/-1.0E-30/
IF(T.EQ.TOLD) GO TO 97
CALL SUMT(Y,TOLD,T-TOLD)
TOLD=T
96  CONTINUE
97  CALL FEEDBK(Y,TOLD)
DO 110 I=1,NNZ
IF(I.GT.93) GO TO 401
PSI(I)=Y(1,I)
401 CONTINUE
DY(I)=0.0
JBB=JB(I)
JE=JB(I)+JA(I)-1
DO 120 J=JBB,JE
LL=NAME(J)
D-005
D-010
D-015
D-020
D-025
D-035
D-050
D-055
D-065
D-075
D-080
D-085
D-090
D-100
D-110
D-115
D-120
D-125
D-130
D-135
D-140

```


D-145
D-180
D-185
D-195
D-200

```

IF(LL-GT.NNZ) GO TO 120
DY(I)=DY(I)+BIGG(J)*(HINV*Y(2,LL)+C2(LL)*Y(1,LL)+C5(LL)*SUM(LL))+C
11(LL)*BIGG(J)*Y(1,LL)+C4(LL)*BIGH(J)*Y(1,LL)
120 CONTINUE
110 CONTINUE
RETURN
END

```

THIS SUBROUTINE IS REQUIRED BY THE TIME INTEGRATION PACKAGE AND
MUST BE SUPPLIED BY THE USER. ITS PURPOSE IS TO EVALUATE THE J
MATRIX NEEDED WHEN THE CORRECTOR EQUATION IS BEING SOLVED.

```

SUBROUTINE JACMAT(Y,YL,T,HINV,A2,N,NY,EPS,DY,F1,PW)
  REAL*8 C1,C2,C4,C5
  REAL*8 BIGG,BIGGG,BIGH
  INTEGER*2 NAME,JA,JB,NNZ,JC,NUMNP,NUMEL,NELDOF,CONN
  COMMON/GTRY2/NAME{4615},JA{183},JB{184},NNZ,JC,NUMNP,NUMEL,NELDOF,
1  CONN{128,15}
  DIMENSION Y(7,1),YL(1),F1(1),DY(1),PW(1)
  COMMON/COEF/C1{183},C2{183},C4{183},C5{183}
  COMMON/SYSMT1/BIGG(4615)
  COMMON/SYSMT2/BIGGG(4615)
  COMMON/SYSMT3/BIGH(4615)
  AH=-A2*HINV
  DO 300 I=1,NNZ
    JBB=JB(I)
    JE=JB(I)+JA(I)-1
    DO 310 J=JBB,JE
      LL=NAME(J)
      PW(J)=BIGG(J)*(AH+C2(LL))+C1(LL)*BIGGG(J)+C4(LL)*BIGH(J)
310 CONTINUE
300 CONTINUE
  RETURN
  END

```

U-115
U-120
U-130
U-135

```
C-----C  
C  
C THIS SUBROUTINE IS REQUIRED BY THE TIME INTEGRATION PACKAGE AND  
C MUST BE SUPPLIED BY THE USER. ITS PURPOSE IS TO SOLVE A LINEAR  
C SYSTEM OF EQUATIONS FOR THE NEWTON ITERATES WHEN THE CORRECTOR  
C EQUATION IS BEING SOLVED.  
C-----C
```

SUBROUTINE NUTSL(PW,DY,FL,N,VY,EPS,YMAX,NEWPW,KRET)
INTEGER*2 K,JA,JB,NNZ,JC,NUMNP,NUMEL,NELDOF,CONN
N-005
N-010


```

COMMON/GTRY2/((4615),JA(183),JB(184),VVZ,JC,NUMNP,NUMEL,NELDOF,CON
1N(128,15)
DIMENSION PW(1),DY(1),F1(1),YMAX(1)
DATA SPD,SPDM1/1.05,.05/
KRET = 0
EPSS = EPSS**2
EPSA2 = EPSS*.0001
NOIT = N
280 DO 281 I=1,NY
    LL=JB(I)
281 F1(I)=DY(I)/PW(LL)
    DO 287 IT=1,NOIT
        RCH = 0.
        CH = 0.
        DO 285 I=1,NY
            LL=JB(I)
            JBB=JB(I)+1
            JE=JB(I)+JA(I)-1
            FN = DY(I)
            DO 284 J=JBB,JE
                IF(K(J).LE.0.OR.K(J).GT.NY) GO TO 284
                FN=FN-PW(J)*F1(K(J))
                CONTINUE
284 FN=FN/PW(LL)
                FN = FN*SPD - SPDM1*F1(I)
                ACH = F1(I) - FN
                CH = CH + (ACH/YMAX(I))**2
282 RCH = RCH + (ACH/AMAX1(ABS(FN),EPS))**2
285 F1(I) = FN
                IF(RCH.LT.EPSS)GO TO 288
                IF(CH.LE.EPSA2)GO TO 288
                CONTINUE
287 KRET=3
288 CONTINUE
    RETURN
END

```

```

C-----
C THIS SUBROUTINE CALCULATES THE GGG ELEMENT MATRIX USING 21 POINTS
C OF INTEGRATION. IT THEN PUTS THE GGG ELEMENT MATRIX INTO THE BIGH
C SYSTEM MATRIX.
C-----
C
C
C
C

```

```

SUBROUTINE FEEDBK(Y,TOLD)
REAL*8 GGG,BIGH
REAL*8 WL,WS,S,CL1,CL2,CL3,CL4,CL5,CL6,CL7,CL8,CL9,CL10,CL11,CL12,CL13,CL14,CL15,CL16,CL17,CL18,CL19,CL20,CL21,CL22,CL23,CL24,CL25,CL26,CL27,CL28,CL29,CL30,CL31,CL32,CL33,CL34,CL35,CL36,CL37,CL38,CL39,CL40,CL41,CL42,CL43,CL44,CL45,CL46,CL47,CL48,CL49,CL50,CL51,CL52,CL53,CL54,CL55,CL56,CL57,CL58,CL59,CL60,CL61,CL62,CL63,CL64,CL65,CL66,CL67,CL68,CL69,CL70,CL71,CL72,CL73,CL74,CL75,CL76,CL77,CL78,CL79,CL80,CL81,CL82,CL83,CL84,CL85,CL86,CL87,CL88,CL89,CL90,CL91,CL92,CL93,CL94,CL95,CL96,CL97,CL98,CL99,CL100,CL101,CL102,CL103,CL104,CL105,CL106,CL107,CL108,CL109,CL110,CL111,CL112,CL113,CL114,CL115,CL116,CL117,CL118,CL119,CL120,CL121,CL122,CL123,CL124,CL125,CL126,CL127,CL128,CL129,CL130,CL131,CL132,CL133,CL134,CL135,CL136,CL137,CL138,CL139,CL140,CL141,CL142,CL143,CL144,CL145,CL146,CL147,CL148,CL149,CL150,CL151,CL152,CL153,CL154,CL155,CL156,CL157,CL158,CL159,CL160,CL161,CL162,CL163,CL164,CL165,CL166,CL167,CL168,CL169,CL170,CL171,CL172,CL173,CL174,CL175,CL176,CL177,CL178,CL179,CL180,CL181,CL182,CL183,CL184,CL185,CL186,CL187,CL188,CL189,CL190,CL191,CL192,CL193,CL194,CL195,CL196,CL197,CL198,CL199,CL200,CL201,CL202,CL203,CL204,CL205,CL206,CL207,CL208,CL209,CL210,CL211,CL212,CL213,CL214,CL215,CL216,CL217,CL218,CL219,CL220,CL221,CL222,CL223,CL224,CL225,CL226,CL227,CL228,CL229,CL230,CL231,CL232,CL233,CL234,CL235,CL236,CL237,CL238,CL239,CL240,CL241,CL242,CL243,CL244,CL245,CL246,CL247,CL248,CL249,CL250,CL251,CL252,CL253,CL254,CL255,CL256,CL257,CL258,CL259,CL260,CL261,CL262,CL263,CL264,CL265,CL266,CL267,CL268,CL269,CL270,CL271,CL272,CL273,CL274,CL275,CL276,CL277,CL278,CL279,CL280,CL281,CL282,CL283,CL284,CL285,CL286,CL287,CL288,CL289,CL290,CL291,CL292,CL293,CL294,CL295,CL296,CL297,CL298,CL299,CL300,CL301,CL302,CL303,CL304,CL305,CL306,CL307,CL308,CL309,CL310,CL311,CL312,CL313,CL314,CL315,CL316,CL317,CL318,CL319,CL320,CL321,CL322,CL323,CL324,CL325,CL326,CL327,CL328,CL329,CL330,CL331,CL332,CL333,CL334,CL335,CL336,CL337,CL338,CL339,CL340,CL341,CL342,CL343,CL344,CL345,CL346,CL347,CL348,CL349,CL350,CL351,CL352,CL353,CL354,CL355,CL356,CL357,CL358,CL359,CL360,CL361,CL362,CL363,CL364,CL365,CL366,CL367,CL368,CL369,CL370,CL371,CL372,CL373,CL374,CL375,CL376,CL377,CL378,CL379,CL380,CL381,CL382,CL383,CL384,CL385,CL386,CL387,CL388,CL389,CL390,CL391,CL392,CL393,CL394,CL395,CL396,CL397,CL398,CL399,CL400,CL401,CL402,CL403,CL404,CL405,CL406,CL407,CL408,CL409,CL410,CL411,CL412,CL413,CL414,CL415,CL416,CL417,CL418,CL419,CL420,CL421,CL422,CL423,CL424,CL425,CL426,CL427,CL428,CL429,CL430,CL431,CL432,CL433,CL434,CL435,CL436,CL437,CL438,CL439,CL440,CL441,CL442,CL443,CL444,CL445,CL446,CL447,CL448,CL449,CL450,CL451,CL452,CL453,CL454,CL455,CL456,CL457,CL458,CL459,CL460,CL461,CL462,CL463,CL464,CL465,CL466,CL467,CL468,CL469,CL470,CL471,CL472,CL473,CL474,CL475,CL476,CL477,CL478,CL479,CL480,CL481,CL482,CL483,CL484,CL485,CL486,CL487,CL488,CL489,CL490,CL491,CL492,CL493,CL494,CL495,CL496,CL497,CL498,CL499,CL500,CL501,CL502,CL503,CL504,CL505,CL506,CL507,CL508,CL509,CL510,CL511,CL512,CL513,CL514,CL515,CL516,CL517,CL518,CL519,CL520,CL521,CL522,CL523,CL524,CL525,CL526,CL527,CL528,CL529,CL530,CL531,CL532,CL533,CL534,CL535,CL536,CL537,CL538,CL539,CL540,CL541,CL542,CL543,CL544,CL545,CL546,CL547,CL548,CL549,CL550,CL551,CL552,CL553,CL554,CL555,CL556,CL557,CL558,CL559,CL560,CL561,CL562,CL563,CL564,CL565,CL566,CL567,CL568,CL569,CL570,CL571,CL572,CL573,CL574,CL575,CL576,CL577,CL578,CL579,CL580,CL581,CL582,CL583,CL584,CL585,CL586,CL587,CL588,CL589,CL590,CL591,CL592,CL593,CL594,CL595,CL596,CL597,CL598,CL599,CL600,CL601,CL602,CL603,CL604,CL605,CL606,CL607,CL608,CL609,CL610,CL611,CL612,CL613,CL614,CL615,CL616,CL617,CL618,CL619,CL620,CL621,CL622,CL623,CL624,CL625,CL626,CL627,CL628,CL629,CL630,CL631,CL632,CL633,CL634,CL635,CL636,CL637,CL638,CL639,CL640,CL641,CL642,CL643,CL644,CL645,CL646,CL647,CL648,CL649,CL650,CL651,CL652,CL653,CL654,CL655,CL656,CL657,CL658,CL659,CL660,CL661,CL662,CL663,CL664,CL665,CL666,CL667,CL668,CL669,CL670,CL671,CL672,CL673,CL674,CL675,CL676,CL677,CL678,CL679,CL680,CL681,CL682,CL683,CL684,CL685,CL686,CL687,CL688,CL689,CL690,CL691,CL692,CL693,CL694,CL695,CL696,CL697,CL698,CL699,CL700,CL701,CL702,CL703,CL704,CL705,CL706,CL707,CL708,CL709,CL710,CL711,CL712,CL713,CL714,CL715,CL716,CL717,CL718,CL719,CL720,CL721,CL722,CL723,CL724,CL725,CL726,CL727,CL728,CL729,CL730,CL731,CL732,CL733,CL734,CL735,CL736,CL737,CL738,CL739,CL740,CL741,CL742,CL743,CL744,CL745,CL746,CL747,CL748,CL749,CL750,CL751,CL752,CL753,CL754,CL755,CL756,CL757,CL758,CL759,CL760,CL761,CL762,CL763,CL764,CL765,CL766,CL767,CL768,CL769,CL770,CL771,CL772,CL773,CL774,CL775,CL776,CL777,CL778,CL779,CL780,CL781,CL782,CL783,CL784,CL785,CL786,CL787,CL788,CL789,CL790,CL791,CL792,CL793,CL794,CL795,CL796,CL797,CL798,CL799,CL800,CL801,CL802,CL803,CL804,CL805,CL806,CL807,CL808,CL809,CL810,CL811,CL812,CL813,CL814,CL815,CL816,CL817,CL818,CL819,CL820,CL821,CL822,CL823,CL824,CL825,CL826,CL827,CL828,CL829,CL830,CL831,CL832,CL833,CL834,CL835,CL836,CL837,CL838,CL839,CL840,CL841,CL842,CL843,CL844,CL845,CL846,CL847,CL848,CL849,CL850,CL851,CL852,CL853,CL854,CL855,CL856,CL857,CL858,CL859,CL860,CL861,CL862,CL863,CL864,CL865,CL866,CL867,CL868,CL869,CL870,CL871,CL872,CL873,CL874,CL875,CL876,CL877,CL878,CL879,CL880,CL881,CL882,CL883,CL884,CL885,CL886,CL887,CL888,CL889,CL890,CL891,CL892,CL893,CL894,CL895,CL896,CL897,CL898,CL899,CL900,CL901,CL902,CL903,CL904,CL905,CL906,CL907,CL908,CL909,CL910,CL911,CL912,CL913,CL914,CL915,CL916,CL917,CL918,CL919,CL920,CL921,CL922,CL923,CL924,CL925,CL926,CL927,CL928,CL929,CL930,CL931,CL932,CL933,CL934,CL935,CL936,CL937,CL938,CL939,CL940,CL941,CL942,CL943,CL944,CL945,CL946,CL947,CL948,CL949,CL950,CL951,CL952,CL953,CL954,CL955,CL956,CL957,CL958,CL959,CL960,CL961,CL962,CL963,CL964,CL965,CL966,CL967,CL968,CL969,CL970,CL971,CL972,CL973,CL974,CL975,CL976,CL977,CL978,CL979,CL980,CL981,CL982,CL983,CL984,CL985,CL986,CL987,CL988,CL989,CL990,CL991,CL992,CL993,CL994,CL995,CL996,CL997,CL998,CL999,CL1000,CL1001,CL1002,CL1003,CL1004,CL1005,CL1006,CL1007,CL1008,CL1009,CL1010,CL1011,CL1012,CL1013,CL1014,CL1015,CL1016,CL1017,CL1018,CL1019,CL1020,CL1021,CL1022,CL1023,CL1024,CL1025,CL1026,CL1027,CL1028,CL1029,CL1030,CL1031,CL1032,CL1033,CL1034,CL1035,CL1036,CL1037,CL1038,CL1039,CL1040,CL1041,CL1042,CL1043,CL1044,CL1045,CL1046,CL1047,CL1048,CL1049,CL1050,CL1051,CL1052,CL1053,CL1054,CL1055,CL1056,CL1057,CL1058,CL1059,CL1060,CL1061,CL1062,CL1063,CL1064,CL1065,CL1066,CL1067,CL1068,CL1069,CL1070,CL1071,CL1072,CL1073,CL1074,CL1075,CL1076,CL1077,CL1078,CL1079,CL1080,CL1081,CL1082,CL1083,CL1084,CL1085,CL1086,CL1087,CL1088,CL1089,CL1090,CL1091,CL1092,CL1093,CL1094,CL1095,CL1096,CL1097,CL1098,CL1099,CL1100,CL1101,CL1102,CL1103,CL1104,CL1105,CL1106,CL1107,CL1108,CL1109,CL1110,CL1111,CL1112,CL1113,CL1114,CL1115,CL1116,CL1117,CL1118,CL1119,CL1120,CL1121,CL1122,CL1123,CL1124,CL1125,CL1126,CL1127,CL1128,CL1129,CL1130,CL1131,CL1132,CL1133,CL1134,CL1135,CL1136,CL1137,CL1138,CL1139,CL1140,CL1141,CL1142,CL1143,CL1144,CL1145,CL1146,CL1147,CL1148,CL1149,CL1150,CL1151,CL1152,CL1153,CL1154,CL1155,CL1156,CL1157,CL1158,CL1159,CL1160,CL1161,CL1162,CL1163,CL1164,CL1165,CL1166,CL1167,CL1168,CL1169,CL1170,CL1171,CL1172,CL1173,CL1174,CL1175,CL1176,CL1177,CL1178,CL1179,CL1180,CL1181,CL1182,CL1183,CL1184,CL1185,CL1186,CL1187,CL1188,CL1189,CL1190,CL1191,CL1192,CL1193,CL1194,CL1195,CL1196,CL1197,CL1198,CL1199,CL1200,CL1201,CL1202,CL1203,CL1204,CL1205,CL1206,CL1207,CL1208,CL1209,CL1210,CL1211,CL1212,CL1213,CL1214,CL1215,CL1216,CL1217,CL1218,CL1219,CL1220,CL1221,CL1222,CL1223,CL1224,CL1225,CL1226,CL1227,CL1228,CL1229,CL1230,CL1231,CL1232,CL1233,CL1234,CL1235,CL1236,CL1237,CL1238,CL1239,CL1240,CL1241,CL1242,CL1243,CL1244,CL1245,CL1246,CL1247,CL1248,CL1249,CL1250,CL1251,CL1252,CL1253,CL1254,CL1255,CL1256,CL1257,CL1258,CL1259,CL1260,CL1261,CL1262,CL1263,CL1264,CL1265,CL1266,CL1267,CL1268,CL1269,CL1270,CL1271,CL1272,CL1273,CL1274,CL1275,CL1276,CL1277,CL1278,CL1279,CL1280,CL1281,CL1282,CL1283,CL1284,CL1285,CL1286,CL1287,CL1288,CL1289,CL1290,CL1291,CL1292,CL1293,CL1294,CL1295,CL1296,CL1297,CL1298,CL1299,CL1300,CL1301,CL1302,CL1303,CL1304,CL1305,CL1306,CL1307,CL1308,CL1309,CL1310,CL1311,CL1312,CL1313,CL1314,CL1315,CL1316,CL1317,CL1318,CL1319,CL1320,CL1321,CL1322,CL1323,CL1324,CL1325,CL1326,CL1327,CL1328,CL1329,CL1330,CL1331,CL1332,CL1333,CL1334,CL1335,CL1336,CL1337,CL1338,CL1339,CL1340,CL1341,CL1342,CL1343,CL1344,CL1345,CL1346,CL1347,CL1348,CL1349,CL1350,CL1351,CL1352,CL1353,CL1354,CL1355,CL1356,CL1357,CL1358,CL1359,CL1360,CL1361,CL1362,CL1363,CL1364,CL1365,CL1366,CL1367,CL1368,CL1369,CL1370,CL1371,CL1372,CL1373,CL1374,CL1375,CL1376,CL1377,CL1378,CL1379,CL1380,CL1381,CL1382,CL1383,CL1384,CL1385,CL1386,CL1387,CL1388,CL1389,CL1390,CL1391,CL1392,CL1393,CL1394,CL1395,CL1396,CL1397,CL1398,CL1399,CL1400,CL1401,CL1402,CL1403,CL1404,CL1405,CL1406,CL1407,CL1408,CL1409,CL1410,CL1411,CL1412,CL1413,CL1414,CL1415,CL1416,CL1417,CL1418,CL1419,CL1420,CL1421,CL1422,CL1423,CL1424,CL1425,CL1426,CL1427,CL1428,CL1429,CL1430,CL1431,CL1432,CL1433,CL1434,CL1435,CL1436,CL1437,CL1438,CL1439,CL1440,CL1441,CL1442,CL1443,CL1444,CL1445,CL1446,CL1447,CL1448,CL1449,CL1450,CL1451,CL1452,CL1453,CL1454,CL1455,CL1456,CL1457,CL1458,CL1459,CL1460,CL1461,CL1462,CL1463,CL1464,CL1465,CL1466,CL1467,CL1468,CL1469,CL1470,CL1471,CL1472,CL1473,CL1474,CL1475,CL1476,CL1477,CL1478,CL1479,CL1480,CL1481,CL1482,CL1483,CL1484,CL1485,CL1486,CL1487,CL1488,CL1489,CL1490,CL1491,CL1492,CL1493,CL1494,CL1495,CL1496,CL1497,CL1498,CL1499,CL1500,CL1501,CL1502,CL1503,CL1504,CL1505,CL1506,CL1507,CL1508,CL1509,CL1510,CL1511,CL1512,CL1513,CL1514,CL1515,CL1516,CL1517,CL1518,CL1519,CL1520,CL1521,CL1522,CL1523,CL1524,CL1525,CL1526,CL1527,CL1528,CL1529,CL1530,CL1531,CL1532,CL1533,CL1534,CL1535,CL1536,CL1537,CL1538,CL1539,CL1540,CL1541,CL1542,CL1543,CL1544,CL1545,CL1546,CL1547,CL1548,CL1549,CL1550,CL1551,CL1552,CL1553,CL1554,CL1555,CL1556,CL1557,CL1558,CL1559,CL1560,CL1561,CL1562,CL1563,CL1564,CL1565,CL1566,CL1567,CL1568,CL1569,CL1570,CL1571,CL1572,CL1573,CL1574,CL1575,CL1576,CL1577,CL1578,CL1579,CL1580,CL1581,CL1582,CL1583,CL1584,CL1585,CL1586,CL1587,CL1588,CL1589,CL1590,CL1591,CL1592,CL1593,CL1594,CL1595,CL1596,CL1597,CL1598,CL1599,CL1600,CL1601,CL1602,CL1603,CL1604,CL1605,CL1606,CL1607,CL1608,CL1609,CL1610,CL1611,CL1612,CL1613,CL1614,CL1615,CL1616,CL1617,CL1618,CL1619,CL1620,CL1621,CL1622,CL1623,CL1624,CL1625,CL1626,CL1627,CL1628,CL1629,CL1630,CL1631,CL1632,CL1633,CL1634,CL1635,CL1636,CL1637,CL1638,CL1639,CL1640,CL1641,CL1642,CL1643,CL1644,CL1645,CL1646,CL1647,CL1648,CL1649,CL1650,CL1651,CL1652,CL1653,CL1654,CL1655,CL1656,CL1657,CL1658,CL1659,CL1660,CL1661,CL1662,CL1663,CL1664,CL1665,CL1666,CL1667,CL1668,CL1669,CL1670,CL1671,CL1672,CL1673,CL1674,CL1675,CL1676,CL1677,CL1678,CL1679,CL1680,CL1681,CL1682,CL1683,CL1684,CL1685,CL1686,CL1687,CL1688,CL1689,CL1690,CL1691,CL1692,CL1693,CL1694,CL1695,CL1696,CL1697,CL1698,CL1699,CL1700,CL1701,CL1702,CL1703,CL1704,CL1705,CL1706,CL1707,CL1708,CL1709,CL1710,CL1711,CL1712,CL1713,CL1714,CL1715,CL1716,CL1717,CL1718,CL1719,CL1720,CL1721,CL1722,CL1723,CL1724,CL1725,CL1726,CL1727,CL1728,CL1729,CL1730,CL1731,CL1732,CL1733,CL1734,CL1735,CL1736,CL1737,CL1738,CL1739,CL1740,CL1741,CL1742,CL1743,CL1744,CL1745,CL1746,CL1747,CL1748,CL1749,CL1750,CL1751,CL1752,CL1753,CL1754,CL1755,CL1756,CL1757,CL1758,CL1759,CL1760,CL1761,CL1762,CL1763,CL1764,CL1765,CL1766,CL1767,CL1768,CL1769,CL1770,CL1771,CL1772,CL1773,CL1774,CL1775,CL1776,CL1777,CL1778,CL1779,CL1780,CL1781,CL1782,CL1783,CL1784,CL1785,CL1786,CL1787,CL1788,CL1789,CL1790,CL1791,CL1792,CL1793,CL1794,CL1795,CL1796,CL1797,CL1798,CL1799,CL1800,CL1801,CL1802,CL1803,CL1804,CL1805,CL1806,CL1807,CL1808,CL1809,CL1810,CL1811,CL1812,CL1813,CL1814,CL1815,CL1816,CL1817,CL1818,CL1819,CL1820,CL1821,CL1822,CL1823,CL1824,CL1825,CL1826,CL1827,CL1828,CL1829,CL1830,CL1831,CL1832,CL1833,CL1834,CL1835,CL1836,CL1837,CL1838,CL1839,CL1840,CL1841,CL1842,CL1843,CL1844,CL1845,CL1846,CL1847,CL1848,CL1849,CL1850,CL1851,CL1852,CL1853,CL1854,CL1855,CL1856,CL1857,CL1858,CL1859,CL1860,CL1861,CL1862,CL1863,CL1864,CL1865,CL1866,CL1867,CL1868,CL1869,CL1870,CL1871,CL1872,CL1873,CL1874,CL1875,CL1876,CL1877,CL1878,CL1879,CL1880,CL1881,CL1882,CL1883,CL1884,CL1885,CL1886,CL1887,CL1888,CL1889,CL1890,CL1891,CL1892,CL1893,CL1894,CL1895,CL1896,CL1897,CL1898,CL1899,CL1900,CL1901,CL1902,CL1903,CL1904,CL1905,CL1906,CL1907,CL1908,CL1909,CL1910,CL1911,CL1912,CL1913,CL1914,CL1915,CL1916,CL1917,CL1918,CL1919,CL1920,CL1921,CL1922,CL1923,CL1924,CL1925,CL1926,CL1927,CL1928,CL1929,CL1930,CL1931,CL1932,CL1933,CL1934,CL1935,CL1936,CL1937,CL1938,CL1939,CL1940,CL1941,CL1942,CL1943,CL1944,CL1945,CL1946,CL1947,CL1948,CL1949,CL1950,CL1951,CL1952,CL1953,CL1954,CL1955,CL1956,CL1957,CL1958,CL1959,CL1960,CL1961,CL1962,CL1963,CL1964,CL1965,CL1966,CL1967,CL1968,CL1969,CL1970,CL1971,CL1972,CL1973,CL1974,CL1975,CL1976,CL1977,CL1978,CL1979,CL1980,CL1981,CL1982,CL1983,CL1984,CL1985,CL1986,CL1987,CL1988,CL1989,CL1990,CL1991,CL1992,CL1993,CL1994,CL1995,CL1996,CL1997,CL1998,CL1999,CL2000,CL2001,CL2002,CL2003,CL2004,CL2005,CL2006,CL2007,CL2008,CL2009,CL2010,CL2011,CL2012,CL2013,CL2014,CL2015,CL2016,CL2017,CL2018,CL2019,CL2020,CL2021,CL2022,CL2023,CL2024,CL2025,CL2026,CL2027,CL2028,CL2029,CL2030,CL2031,CL2032,CL2033,CL2034,CL2035,CL2036,CL2037,CL2038,CL2039,CL2040,CL2041,CL2042,CL2043,CL2044,CL2045,CL2046,CL2047,CL2048,CL2049,CL2050,CL2051,CL2052,CL2053,CL2054,CL2055,CL2056,CL2057,CL2058,CL2059,CL2060,CL2061,CL2062,CL2063,CL2064,CL2065,CL2066,CL2067,CL2068,CL2069,CL2070,CL2071,CL2072,CL2073,CL2074,CL2075,CL2076,CL2077,CL2078,CL2079,CL2080,CL2081,CL2082,CL2083,CL2084,CL2085,CL2086,CL2087,CL2088,CL2089,CL2090,CL2091,CL2092,CL2093,CL2094,CL2095,CL2096,CL2097,CL2098,CL2099,CL2100,CL2101,CL2102,CL2103,CL2104,CL2105,CL2106,CL2107,CL2108,CL2109,CL2110,CL2111,CL2112,CL2113,CL2114,CL2115,CL2116,CL2117,CL2118,CL2119,CL2120,CL2121,CL2122,CL2123,CL2124,CL2125,CL2126,CL2127,CL2128,CL2129,CL2130,CL2131,CL2132,CL2133,CL2134,CL2135,CL2136,CL2137,CL2138,CL2139,CL2140,CL2141,CL2142,CL2143,CL2144,CL2145,CL2146
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COMMON/GTRY2/VAME(4615),JA(183),JB(184),NNZ,JC,NUMNP,NUMEL,NELDOF,
1CONN(128,15)
COMMON/GTRY1/X(505),P(505),Z(505)
COMMON/XOCAL/XX(15),YY(15),ZZ(15),PPSI(15)
DIMENSION Y(7,1)
COMMON/TEMP/C3(183),C6(15)
COMMON/VGGGMAT/GGG(15,15)
COMMON/SY SMT3/BIGH(4615)
COMMON/GAUSS/WL(7),WS(5),S(5),CL1(7),CL2(7),CL3(7),FN(15,21),DL1(1
15,21),DL2(15,21),DL3(15,21),DS(15,21),DETJ(21)
DO 300 I=1,JC
BIGH(I)=0.0
CONTINUE
300 NNAAA=7
NNSS=3
DO 800 L=1,NUMEL
DO 600 I=1,15
DO 610 J=1,15
GGG(I,J)=0.0
CONTINUE
610 CONTINUE
DO 805 JJ=1,15
NN=CONN(L,JJ)
XX(JJ)=X(NN)
YY(JJ)=P(NN)
ZZ(JJ)=Z(NN)
IF(NN.GT.NNZ) GO TO 803
C6(JJ)=C3(NN)
PPSI(JJ)=Y(1,NN)
GO TO 805
803 PPSI(JJ)=0.0
C6(JJ)=0.0
CONTINUE
805 A5=2.0/(ZZ(1)-ZZ(15))
DETJJ=0.0
DO 802 K=1,NNSS
DETJL=0.0
DO 798 M=1,NNAAA
IF(K.EQ.1) GO TO 111
IF(K.EQ.2) GO TO 222
IF(K.EQ.3) GO TO 333
GO TO 788
111 N=M
GO TO 788
222 N=M+7
GO TO 788
333 N=M+14
788 CONTINUE

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T-025
T-030
T-035
T-045
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T1=DL1(1,N)*XX(1)+DL1(2,N)*XX(2)-DL3(4,N)*XX(4)-DL3(5,N)*XX(5)
T2=DL1(7,N)*XX(7)-DL3(9,N)*XX(9)+DL1(10,N)*XX(10)+DL1(11,N)*XX(11)
T3=(DL1(6,N)-DL3(6,N))*XX(6)-DL3(13,N)*XX(13)-DL3(14,N)*XX(14)
T4=(DL1(15,N)-DL3(15,N))*XX(15)
DJ1=T1+T2+T3+T4
D1=DL1(1,N)*YY(1)+DL1(2,N)*YY(2)-DL3(4,N)*YY(4)-DL3(5,N)*YY(5)
D2=DL1(7,N)*YY(7)-DL3(9,N)*YY(9)+DL1(10,N)*YY(10)+DL1(11,N)*YY(11)
D3=(DL1(6,N)-DL3(6,N))*YY(6)-DL3(13,N)*YY(13)-DL3(14,N)*YY(14)
D4=(DL1(15,N)-DL3(15,N))*YY(15)
DJ12=D1+D2+D3+D4
T5=DL2(2,N)*XX(2)+DL2(3,N)*XX(3)-DL3(5,N)*XX(5)-DL3(6,N)*XX(6)
T6=DL2(8,N)*XX(8)-DL3(9,N)*XX(9)+(DL2(4,N)-DL3(4,N))*XX(4)
T7=-DL2(11,N)*XX(11)+DL2(12,N)*XX(12)+(DL2(13,N)-DL3(13,N))*XX(13)
T8=-DL3(14,N)*XX(14)-DL3(15,N)*XX(15)
DJ21=T5+T6+T7+T8
D5=DL2(2,N)*YY(2)+DL2(3,N)*YY(3)-DL3(5,N)*YY(5)-DL3(6,N)*YY(6)
D6=DL2(8,N)*YY(8)-DL3(9,N)*YY(9)+(DL2(4,N)-DL3(4,N))*YY(4)
D7=DL2(11,N)*YY(11)+DL2(12,N)*YY(12)+(DL2(13,N)-DL3(13,N))*YY(13)
D8=-DL3(14,N)*YY(14)-DL3(15,N)*YY(15)
DJ22=D5+D6+D7+D8
DETJ(N)=DJ11*DJ22-DJ12*DJ21
DETJ1=DETJ(N)*WL(M)+DETJ1
CONTINUE
DETJJ=DETJJ+WS(K)*DETJ1
CONTINUE
DO 700 J=1,15
DO 710 I=1,15
FHH=0.0
DO 806 K=1,NNSSS
FH=0.0
DO 799 M=1,NNAAA
TT1=0.0
TT2=0.0
TT3=0.0
IF(K.EQ.1) GO TO 1111
IF(K.EQ.2) GO TO 2222
IF(K.EQ.3) GO TO 3333
GO TO 7888
1111 N=M
GO TO 7888
2222 N=M+7
GO TO 7888
3333 N=M+14
7888 CONTINUE
TT1=FN(1,N)*PPSI(1)*C6(1)+FN(2,N)*PPSI(2)*C6(2)+FN(3,N)*PPSI(3)*C6
1(3)+FN(4,N)*PPSI(4)*C6(4)+FN(5,N)*PPSI(5)*C6(5)+FN(6,N)*PPSI(6)*C6
1(6)+FN(7,N)*PPSI(7)*C6(7)+FN(8,N)*PPSI(8)*C6(8)
TT2=FN(9,N)*PPSI(9)*C6(9)+FN(10,N)*PPSI(10)*C6(10)+FN(11,N)*PPSI(1

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T-550

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798
802

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1111
2222
3333
7888
770

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11)*C6(11)+FN(12,N)*PPSI(12)*C6(12)+FN(13,N)*PPSI(13)*C6(13)+FN(14,
1N)*PPSI(14)*C6(14)+FN(15,N)*PPSI(15)*C6(15)
TT3=TT1+TT2+1.0
FH=FH+WL(M)*FN(J,N)*FN(I,N)*ALOG(TT3)*DETJ(N)
799 CONTINUE
806 FHH=WS(K)*FH+FHH
CONTINUE
GGG(J,I)=FHH/A5
710 CONTINUE
700 CONTINUE
C INSERT INTO SYSTEM MATRIX
DO 20 K=1,15
KK=CONN(L,K)
IF(KK.GT.NNZ) GO TO 20
KKK=JA(KK)
LLL=JB(KK)-1
DO 10 I=1,15
II=CONN(L,I)
DO 91 M=1,KKK
MM=LLL+M
KKM=NAME(MM)
IF(II.EQ.KKM)GO TO 92
91 CONTINUE
GO TO 10
92 CONTINUE
BIGH(MM)=BIGH(MM)+GGG(K,I)
10 CONTINUE
20 CONTINUE
800 CONTINUE
RETURN
END
C-----
C THE CUMULATIVE CONTRIBUTION OF THE DELAYED NEUTRON FLUX IS
C CALCULATED BY THIS SUBROUTINE.
C-----
C
SUBROUTINE SUMT(Y,T,H)
DIMENSION Y(7,1)
COMMON/DELAY/SUM(183),PSI(93)
Q=0.4349710
TC=-Q*H
DO 100 I=1,93
SUM(I)=SUM(I)*EXP(-Q*H)+0.50*H*(PSI(I)*EXP(TC)+Y(1,I))
100 CONTINUE
RETURN
END
S-005
S-010
S-030
S-040
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S-055
S-060
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T-685
T-690
T-695
T-700
T-705
T-710
T-715
T-720

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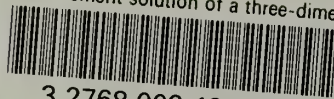
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